Rev. Econ. Design (2011) 15:91–120
DOI 10.1007/s10058-009-0093-8

ORIGINAL PAPER

Countervailing power? Collusion in markets with decentralized trade

Nadia Burani · Clara Ponsati

Received: 20 March 2008 / Accepted: 9 September 2009 / Published online: 22 September 2009

Abstract We consider the collective incentives of buyers and sellers to form cartels in markets with decentralized trade and pairwise bargaining. Cartels are coalitions of buyers or sellers that limit market participation and compensate inactive members for their abstention. In stable market outcomes, cartels set Nash equilibrium quantities and cartel memberships are immune to deviations. The set of stable market outcomes is non-empty and its full characterization is provided. Stable market outcomes are of two types: (i) at least one cartel restrains trade and market participation is balanced; (ii) only one cartel is active and it reduces trade slightly below the opponent's.

Keywords Decentralized trade · Pairwise bargaining · Bilateral cartel formation · Cartel stability · Countervailing power

JEL Classification C78 · D43 · L11

1 Introduction

Collective incentives to restrict trade have long been acknowledged as a prevalent phenomenon in markets. The inherent instability of cooperative agreements attempting to exploit such incentives has also been extensively discussed in the literature.

N. Burani (✉)
Dipartimento di Scienze Economiche, Università di Bologna,
Strada Maggiore 45, 40125 Bologna, Italy
e-mail: nadia.burani@unibo.it

C. Ponsati
Institut d'Anàlisi Econòmica, CSIC,
Campus Universitat Autònoma de Barcelona UAB, 08193 Bellaterra, Barcelona, Spain
e-mail: clara.ponsati@iae.csic.es

A common feature of the models that address these issues consists in assuming that collusive practices arise only on one side of the market. For instance, particular emphasis has been given to the formation of cartels by oligopolistic firms that face price-taking consumers.[1] More recently, developments in auction theory have addressed the issue of collusion among numerous buyers facing a single seller, or vice versa.[2] Furthermore, new bargaining concepts have been introduced in the literature on industrial relations to study negotiations between labor unions and a firm.[3]

When traders on both sides of a market behave strategically, both sides can in principle form cartels with the purpose of enhancing their collective market power with respect to the opponent's. Is it then possible that cartels emerge and persist on the two sides of a market? Is it possible for collusion to be a desirable phenomenon in this context?

These questions were raised long ago by Galbraith (1952), who claimed positive answers, in his theory of *countervailing power*. Galbraith asserted that "in the competitive model, the power of the firm as a seller is checked or circumscribed by the competitor who offers, or threatens to offer, a better bargain. The role of the buyer on the other side of such market is essentially a passive one. However, (…) in the typical modern market of few sellers, the active restraint is provided not by the competitor but from the other side of the market by strong buyers". [4] Thus, the existence of market power on one side of the market would create an incentive for the other side to organize another position of power neutralizing the former. Countervailing power was seen behind the emergence of labor unions: "One finds the strongest labor unions in the United States where markets are served by strong corporations. And it is not an accident that the large automobile, steel, electrical, farm machinery companies all bargain with powerful unions. It is the strength of the corporations in these industries that made it necessary for workers to develop the protection of countervailing power". [5] The retail market offered another example of the operation of countervailing power. The great development of department-store chains, food chains, mail order houses was interpreted as the countervailing response of retailers, on consumers' behalf, to sellers' previously established positions of power.

Galbraith's claims were not sustained with a rigorous model.[6] And the empirical evidence is somewhat controversial.[7] Yet, the preceding descriptions are suggestive and, despite their formal shortcomings, Galbraith's arguments had great impact on the development of economic policies in the second half of the 20th century. More than fifty years later, very little research has formally addressed the problem. The few

[1] See Donsimoni et al. (1986) and D'Aspremont et al. (1983) for a characterization of stable cartels in the context of oligopolistic markets.

[2] See McAfee and McMillan (1992) and Snyder (1996), respectively.

[3] For analyses of the role played by workers' unions in bargaining with a single firm, see Horn and Wolinsky (1988) and Stole and Zwiebel (1996).

[4] Galbraith (1952, p. 113).

[5] Galbraith (1952, pp. 114–115).

[6] As Stigler (1954) pointed out in his critique.

[7] See Sherer Ross (1990, chap. 14), and references therein.

exceptions are von Ungern-sternberg (1996), Snyder (1996), Inderst and Wey (2003) and Bloch and Ghosal (2000).[8] The first paper develops a theoretical model featuring a single seller simultaneously bargaining on the wholesale price with each one of a finite number of retailers. The retailers will then compete among themselves in the final product market. The main finding is that a decrease in the number of retailers allows them to extract lower prices from their supplier, but increased concentration at the distribution level has the opposing effect of increasing equilibrium consumer prices.[9] Snyder (1996) analyses the same problem in the context of an infinitely repeated procurement auction with one buyer receiving price bids from several sellers. In the spirit of folk theorems for repeated games, he shows that collusive agreements on price bids are most difficult to sustain when the buyer can alter his intertemporal consumption pattern by accumulating a backlog of unfilled orders and by purchasing all at once. Thus a "large" buyer, namely a buyer with a high demand, may gain a strategic advantage over the sellers and obtain a lower price from the sellers. Finally, Inderst and Wey (2003) analyze the effect of horizontal mergers in bilaterally oligopolistic industries on negotiated input prices (and on technology choice by suppliers). Their model describes an intermediary goods market with two producers and two retailers having independent demands, whereby neither the problem of increased concentration in the final product market nor the question of coalitional stability are at stake.

The present work aims at contributing to the game-theoretic scrutiny of the predictions of the theory of countervailing power. As said, its focus is on collusion being both *endogenous* and *bilateral*, and thus on the problem of stability of collusive agreements among buyers and sellers. Such problem is examined in the context of decentralized exchange economies *à la* Rubinstein and Wolinsky (1985), with a continuum of homogeneous buyers and a continuum of homogeneous sellers. In markets that remain stationary at all rounds of trade (as in Rubinstein and Wolinsky 1985, where, at each round, new traders enter exactly in the same measure as satisfied traders exit), the advantage of the short side of the market is not sufficient to create collective incentives on either side to exclude some traders from the market.[10] However, such incentives do exist, and can be strong, in a market that does not remain stationary as it clears over several rounds of trade. Consequently, our analysis is carried out within environments where the relative measure of buyers to sellers changes across the different rounds of trade.

The timing of the model is as follows. In the first stage, buyers form at most one cartel and decide to restrict the measure of traders they put on the market; simultaneously, sellers form at most one cartel, and decide to restrict the measure of traders entering the market. In the second stage, given the cartels' choices about how many members to exclude from trade, all active traders trade on the market at all rounds of trade. Our analysis proceeds by backward induction and starts from the last stage, moving back to the first.

[8] The latter will be discussed later, after the presentation of our results.

[9] Note that the results are based on some comparative statics on the number of retailers. The model does not address the issue of the desirability or sustainability of endogenous collusion among retailers.

[10] See Bloch and Ghosal (2000) for a proof of this claim.

In the last stage, trade in the market is decentralized, since buyers and sellers are randomly matched in pairs and bargain over the price to exchange one unit of an indivisible good. The market operates for finitely many rounds, with no entry of new traders after the first round. At each round, buyers and sellers search for a trading partner and, if they find a match, they bargain over the price at which to transact. If they reach an agreement, they trade and exit the market, otherwise they search again in the following round. Equilibrium prices at the different rounds of trade depend on the relative measures of buyers and sellers that are active in the market at those rounds. In particular, prices are such that agents in the short side of the market are able to apportion a bigger fraction of the surplus generated by trade.

Given that it pays to be in the short side of the market, before the decentralized matching stage, agents have incentive to collude and form cartels that reduce quantities demanded or supplied. In the first stage, we suppose that only one cartel can form on each side of the market and that a cartel might not have control over the whole population on its side, but only on a fraction of it. The only instrument that cartels have at their disposal is the restriction in the market participation of their members.[11] Thus, each cartel chooses how many members, if any, to withdraw from the market and it redistributes its total payoff in order to compensate inactive members for their abstention. Given the indivisibility of the good traded, this is equivalent to cartels setting their own supply or demand. Moreover, since outsiders of the cartel always participate in the market, cartels actually determine the total quantities that will be supplied or demanded in the market. In particular, the two cartels play a non-cooperative game where the quantities supplied and demanded are set simultaneously, taking as given the initial measures of buyers and sellers and the possible sizes of cartel memberships.[12] Non-members generally benefit from the formation of a cartel: they trade the indivisible good at the same, higher price as cartel members, but they do not have to compensate inactive cartel members. This free-riding problem greatly limits the extent to which cartels can effectively reduce trade while expecting to maintain their memberships. Consequently, not all outcomes attained as equilibria of the quantity-setting game are equally relevant.

A natural criterion for selecting among the equilibria of the quantity-setting game consists in requiring that they be supported by stable levels of cartel memberships. In the analysis of cartel formation, we adopt a cooperative rather than a non-cooperative approach. The details of the process that leads to the formation of a cartel of given measure are not specified, nor are the possible actions (participate or not in the cartel) to be taken by the agents. We just state that whenever a cartel forms, its membership

11 Actual cartels have a major impact both on the search and on the bargaining patterns of traders. Cartels may turn a market with decentralized trade into a market with centralized trade, substantially altering the process of price formation. In our model, however, cartels are endowed with much weaker prerogatives. We consider this assumption particularly appropriate to decentralized markets. Observe that we are imposing the minimal requirements on cartels' activity precisely because we want to study whether collusion is sustainable even under the most unfavourable conditions. By so doing, we are abstracting from the problem of the contracting process within cartels (exclusion contracts vs profit sharing contracts without exclusion).

12 This is in contrast with Galbraith's view of countervailing power as a sequential phenomenon; nonetheless, a simultaneous rather than a sequential framework is more tailored to the other features of our model.

must be stable in the following sense: (i) no outsiders unilaterally have incentive to join the cartel; (ii) no insiders unilaterally have incentive to leave the cartel. Therefore, we consider *stable market outcomes*, that is equilibria of the quantity-setting game at which memberships are stable, in that cartel sizes do not trigger deviations. We prove that the set of stable market outcomes is non-empty, and we provide its full characterization. Stable market outcomes can be of two different types.

The first type is such that at least one cartel actively restrains trade and such that the level of participation in the market is balanced on both sides, regardless of the potential sizes of supply and demand. Market outcomes might be inefficient when both cartels are active, because not all gains from trade are apportioned. Thus, using Galbraith's terminology, *both sides exercise countervailing power*. But when only one cartel (the one that forms in the long side of the market) is active, only one side of the market exercises countervailing power, restraining its participation up to the point at which supply and demand coincide. Consequently, this kind of stable market outcome results in an efficient allocation and the effect of countervailing power is limited to a redistribution of the total surplus.

The second type of stable market outcomes is such that only one cartel (more likely the one that forms on the long side of the market) is active, which reduces its participation in the market so as to slightly undercut the opponent's. In this situation, only one side of the market exercises countervailing power and the total surplus is redistributed in favor of this side. The market outcome is not efficient, but the reduction in the quantity traded with respect to its potential total volume is not very significant.

Our paper owes much to Bloch and Ghosal (2000) that precedes us in addressing the issue of cartel formation in the context of an exchange economy with bilateral trade and bargaining. Bloch and Ghosal (2000) consider the formation of cartels of buyers or sellers in markets with an equal and finite number of buyers and sellers. They show that cartels might be active on both sides of the market, but active cartels never withdraw more than one trader from the market. Although there are many apparent differences between our work and that of Bloch and Ghosal (2000), our results are closely related and, we believe, complementary to theirs. The peculiarity of our model can be ascribed to our assumption that cartels set *continuous quantities*. The continuum assumption, although debatable from a descriptive point of view (since bilateral collusion seems more likely in markets with small numbers of traders on each side), is crucial to attain a tractable analysis, and permits to analyze two-sided cartel activity in markets that are *ex ante* unbalanced, a case that is not addressed by Bloch and Ghosal (2000).

The rest of the paper is organized as follows. The basic model of decentralized trade, which describes the last stage of the game, is presented in Sect. 2. The non-cooperative, quantity-setting game played by the cartels is described in Sect. 3, and its equilibria are characterized in Sect. 3.1. The notion of cartel stability is introduced in Sect. 4, where stable market outcomes are characterized.

2 Decentralized trade

Consider a market with a continuum of identical sellers and a continuum of identical buyers. Each seller owns one unit of a homogeneous, indivisible good and his valuation of the good is normalized at zero. Each buyer owns one unit of a perfectly divisible commodity and his valuation for the indivisible good is normalized at one. All agents are perfectly patient.[13]

The market operates for two trading rounds $t = 1, 2$.[14] It is assumed that a Lebesgue measure b of buyers and a Lebesgue measure s of sellers enter the market in the first round. The market is not necessarily balanced, being the measure of buyers potentially different from the measure of sellers. No new agents enter after the first round. In each round, buyers and sellers are randomly matched in pairs and each pair bargain over the surplus generated by the indivisible good.[15] At each round t, traders are randomly matched according to an exogenously given matching function $m_t = m(b_t, s_t)$ that gives the mass of meetings realized at time t as a function of the measure of active buyers b_t and sellers s_t at that round.

There are two basic and very standard conditions that our matching function should satisfy: (i) it should be such that if, for example, the measure of buyers is greater than the measure of sellers, then a buyer's probability of being matched is less than a seller's; (ii) it should exhibit *search frictions* in that, even when the measures of agents on the two sides of the market are equal, it should not generate an exhaustive pairwise matching between all buyers and sellers, so that $m(b_t, s_t) < \min\{b_t, s_t\}$.[16] Given that incentives to restrict trade arise only in non-stationary markets and given that there are no new inflows of traders after the first round, search frictions are needed to ensure the existence of positive measures of agents of each type at each round, and thus to prevent the market from collapsing after the first round of trade.

A Leontief matching function with exogenous frictions is the simplest matching function we could conceive that satisfies the two above mentioned conditions. Therefore, the mass of matches that occur in period t, is assumed to be equal to

$$m_t = \gamma \min\{b_t, s_t\},$$

with $\gamma \in (0, 1)$. As in Wooders (1997), Moreno and Wooders (2002) and Ponsati (2004), the parameter γ is used as a measure of frictions and indexes the efficiency of the random matching process. When the m_t meetings are random, one can compute the probability that a seller finds a partner in period t, which is $\theta_{S,t} = \frac{m_t}{s_t}$, and the

13 Introducing pure time preferences that are common for all agents would not alter the qualitative features of the results.

14 This assumption is made for tractability. The results could in principle be generalized to the case in which the market operates for more trading rounds; but, as the number of periods grows, more and more restrictions on the parameters of the model have to be added in order for the incentives to collude to be maintained.

15 The above description is best interpreted as the stage game of a repeated game, in which buyers and sellers repeatedly want to buy and sell, respectively, one unit of a good that perishes after two rounds of trade.

16 The reader is referred to Lagos (2000), who states that, in the search literature, frictions are "certain features of the environment that prevent some bilateral meetings from taking place". We stick to this traditional approach and take search frictions as a primitive of the model, rather than follow Lagos (2000) and consider frictions as being a property of the equilibrium allocation.

probability that buyer finds a partner in period t, which equals $\theta_{B,t} = \frac{m_t}{b_t}$. In particular, letting sellers be the *short side* of the market at period t, i.e. $s_t < b_t$, then sellers do not find a trading partner with certainty but only with probability $\theta_{S,t} = \gamma$, while buyers are matched with an even smaller probability $\theta_{B,t} = \gamma \frac{s_t}{b_t}$.

When a buyer and a seller get matched, they bargain on a price to trade the indivisible object. At either round of trade, the bargaining game consists in an ultimatum game. Namely, a fair lottery selects one of the parties to propose a partition of the surplus; the other party responds by accepting the offer or by rejecting it. Upon acceptance, the agents trade and leave the market. Upon rejection in the first period, the match breaks and the agents return to the market, searching for other partners in the second round of trade. A rejection in the second round implies that the game ends without trade for the given match. The payoffs for trading at price $p \in [0, 1]$ at either round are p to the seller and $1 - p$ to the buyer. The utility associated with no trade is zero.

At the unique subgame perfect equilibrium of the bargaining game, the proposer offers the responder a share equal to the latter's expected value of returning to the pool of unmatched agents in the following round, and the responder accepts. In other words, the responder is given his *outside option* at that period and the proposer gets the residual surplus. Consequently, there exists a unique market equilibrium such that, at each round, all pairs of traders immediately agree on the same price.[17] Since the populations of buyers and sellers typically change from one round to the next, the bargaining pairs face endogenous and time-varying outside options.

Suppose that sellers are initially the short side of the market, i.e. $s < b$. At period t, outside options can be defined recursively as

$$x^L_{B,t} = \theta_{B,t+1}\left(\frac{1}{2}\left(1 - x^S_{S,t+1}\right) + \frac{1}{2}x^L_{B,t+1}\right) + \left(1 - \theta_{B,t+1}\right)x^L_{B,t+1}$$

for buyers, and

$$x^S_{S,t} = \theta_{S,t+1}\left(\frac{1}{2}\left(1 - x^L_{B,t+1}\right) + \frac{1}{2}x^S_{S,t+1}\right) + \left(1 - \theta_{S,t+1}\right)x^S_{S,t+1}$$

for sellers, where the superscripts L and S stand for *long* and *short side* of the market, respectively. When there are only two trading rounds, starting from the second period and substituting backwards, one can compute the agents' present discounted value of participating in a two-rounds market, which is

$$\pi^L_B \equiv x^L_{B,0} = \frac{\left(\theta_{B,1}\left(2 - \theta_{S,2}\right) + \left(2 - \theta_{B,1}\right)\theta_{B,2}\right)}{4} \tag{1}$$

[17] The assumption that traders use a (random proposer) ultimatum game is made for expositional clarity. Other bargaining games yield the same market equilibrium. See Ponsati (2004) for a proof that under infinite horizon alternating offers bargaining, ultimatum strategies can still prevail and yield a unique market equilibrium as well.

for each buyer, and

$$\pi_S^S \equiv x_{S,0}^S = \frac{\left(\theta_{S,1}\left(2-\theta_{B,2}\right)+\left(2-\theta_{S,1}\right)\theta_{S,2}\right)}{4} \tag{2}$$

for each seller. Note that both π_B^L and π_S^S depend on the initial measures of buyers b and sellers s, through the matching probabilities $\theta_{B,1} = \gamma \frac{s}{b}$ and $\theta_{B,2} = \gamma \frac{(1-\gamma)s}{b-\gamma s}$, while $\theta_{S,t} = \gamma$ for every t. Symmetric expected values can be computed if $b \leq s$. Thus, in general, substituting into (1) and (2) the expressions for the matching probabilities, one can write the *individual expected utility* to a buyer, when the initial measure of buyers is b and the initial measure of sellers is s, as

$$\pi_B(b,s) = \begin{cases} \pi_B^S(b,s) = \gamma \dfrac{(4-\gamma)(s-\gamma b)-(1-\gamma)\gamma b}{4(s-\gamma b)} & \text{if } b < s \\ \pi_B^L(b,s) = \gamma \dfrac{s}{b} \dfrac{(3-2\gamma)(b-\gamma s)+(1-\gamma) b}{4(b-\gamma s)} & \text{if } b \geq s \end{cases}, \tag{3}$$

where $\pi_B^S(b,s) = \pi_B^L(b,s)$ for $b = s$. Symmetrically, the *individual expected utility* to a seller is

$$\pi_S(b,s) = \begin{cases} \pi_S^S(b,s) = \gamma \dfrac{(4-\gamma)(b-\gamma s)-(1-\gamma)\gamma s}{4(b-\gamma s)} & \text{if } s < b \\ \pi_S^L(b,s) = \gamma \dfrac{b}{s} \dfrac{(3-2\gamma)(s-\gamma b)+(1-\gamma) s}{4(s-\gamma b)} & \text{if } s \geq b \end{cases}. \tag{4}$$

As for the *group expected payoff* of traders on the same side of the market, there might exist collective incentives to exclude some agents from trade. This occurs because buyers' (sellers') collective utility generally increases when the measure of buyers (sellers) trading on the market is reduced, given the measure of active sellers (buyers).

The group expected payoff to the buyers that enter the market in measure b and face a measure s of sellers is simply $\pi_B(b,s)b$. When the buyers are the long side of the market, that is when $b \geq s$, their joint utility is

$$\pi_B^L(b,s)b = \gamma s \frac{(3-2\gamma)(b-\gamma s)+(1-\gamma) b}{4(b-\gamma s)}.$$

Taking the derivative of $\pi_B^L(b,s)b$ with respect to b yields

$$\frac{\partial}{\partial b}\pi_B^L(b,s)b = -\frac{(1-\gamma)\gamma^2 s^2}{4(b-\gamma s)^2} < 0.$$

It immediately follows that a decrease in the measure of active traders is always collectively beneficial for the long side of the market. Conversely, when $b < s$, the collective utility of buyers in the short side of the market is

$$\pi_B^S(b,s)b = b\gamma \frac{(4-\gamma)(s-\gamma b)-(1-\gamma)\gamma b}{4(s-\gamma b)}.$$

It is easy to check that $\pi_B^S(b,s)b$ is strictly concave in b in all its domain and that it reaches a maximum at

$$\widehat{b}(s) \equiv s\frac{(5-2\gamma)-\sqrt{(1-\gamma)(5-2\gamma)}}{\gamma(5-2\gamma)}.$$

Note that the above maximum lies outside the relevant range when $\widehat{b}(s) \geq s$, that is when search frictions satisfy $\gamma \leq \frac{7-\sqrt{17}}{4} = 0.71922$.[18]

Precisely the same conclusions hold for the sellers' group payoffs.

Therefore, collective incentives to restrict market participation are present, and they might exist even for the short side of the market. It seems then natural to analyze whether coalitions that attempt to restrict trade are sustainable or not, and what impact they have on market performance. We address these issues in the following sections.

3 Cartel games

In the first stage, the timing of the game is such that buyers form at most one cartel, and the buyers' cartel chooses the measure of members to withdraw from the market. At the same time, sellers form at most one cartel, and the sellers' cartel chooses the measure of its active members. Despite the simultaneous nature of these decisions, our exposition is such that the analysis of cartel formation is separated from the analysis of the cartels' reducing the quantity traded. Indeed, in the present section, cartel memberships are considered as exogenous parameters: cartel sizes are taken as given, as if the problem of cartel formation had already been solved in a previous stage; only in Sect. 4 will cartel sizes be endogenized.

We assume that one and only one cartel operates on each side of the market.[19] The buyers' cartel controls a measure $\mu_B b$ of buyers, and a mass $\mu_S s$ of sellers belong to the sellers' cartel, where $0 < \mu_B, \mu_S \leq 1$. There are $(1-\mu_B)\, b$ free buyers and $(1-\mu_S)\, s$ free sellers, who operate as independent traders and remain active in the market as long as they have trades to carry out. Outsiders to the cartel do not play strategically: it is common knowledge that they always participate in the decentralized market and are always willing to trade according to the mechanisms outlined in Sect. 2.

Cartels play a *quantity-setting game* where they simultaneously choose their participation level in the market. Given the bilateral nature of trade and the indivisibility of the traded good, each cartel restricts the quantity actually traded by withdrawing

[18] See Sect. 3.1 for further reference.

[19] The coalition structure formed by only one cartel and many independent traders is taken as a primitive of the model. Nonetheless, it is based on Ray and Vohra (1999) who prove that, in a standard symmetric Cournot oligopoly with n firms, the only stable coalition structure is one with a single cartel and the remaining firms as singletons.

some measure of its members from the market. In particular, the buyers' cartel sets the measure of its active members, i.e. it sets its demand $q_B \in [0, \mu_B b]$, and the sellers' cartel sets its supply $q_S \in [0, \mu_S s]$. Since free traders always participate in trade, the total quantity demanded in the market is given by $\beta = q_B + (1 - \mu_B) b$ and the total quantity supplied in the market is equal to $\sigma = q_S + (1 - \mu_S) s$. Therefore, it is as if the buyers' cartel chooses market demand $\beta \in [(1 - \mu_B) b, b]$ and the sellers' cartel determines market supply $\sigma \in [(1 - \mu_S) s, s]$.

The aggregate cartel payoffs are then redistributed equally among cartel members in order to compensate the inactive agents for their abstention.[20]

For each given profile of cartel memberships $\mu = (\mu_B, \mu_S)$, the cartels' payoffs are derived from (3) and (4), respectively, and can be expressed as functions of market demand and supply as follows

$$\Pi_B(\beta,\sigma) = \begin{cases} (\beta-(1-\mu_B)b)\,\pi_B^S(\beta,\sigma) = \dfrac{\gamma(\beta-(1-\mu_B)b)((4-\gamma)(\sigma-\gamma\beta)-(1-\gamma)\gamma\beta)}{4(\sigma-\gamma\beta)} \\ \quad \text{if } (1-\mu_B)b \le \beta < \sigma \\ (\beta-(1-\mu_B)b)\,\pi_B^L(\beta,\sigma) = \dfrac{\gamma\sigma(\beta-(1-\mu_B)b)((3-2\gamma)(\beta-\gamma\sigma)+(1-\gamma)\beta)}{4\beta(\beta-\gamma\sigma)} \\ \quad \text{if } \sigma \le \beta \le b \end{cases}$$

and

$$\Pi_S(\beta,\sigma) = \begin{cases} (\sigma-(1-\mu_S)s)\,\pi_S^S(\beta,\sigma) = \dfrac{\gamma(\sigma-(1-\mu_S)s)((4-\gamma)(\beta-\gamma\sigma)-(1-\gamma)\gamma\sigma)}{4(\beta-\gamma\sigma)} \\ \quad \text{if } (1-\mu_S)s \le \sigma < \beta \\ (\sigma-(1-\mu_S)s)\,\pi_S^L(\beta,\sigma) = \dfrac{\gamma\beta(\sigma-(1-\mu_S)s)((3-2\gamma)(\sigma-\gamma\beta)+(1-\gamma)\sigma)}{4\sigma(\sigma-\gamma\beta)} \\ \quad \text{if } \beta \le \sigma \le s \end{cases}.$$

The buyers' cartel faces the problem $\max_\beta \Pi_B(\beta,\sigma)$ and its solution is the correspondence $\beta(\sigma)$ which associates the profit-maximizing quantities set by the buyers' cartel to any level of market supply σ, and similarly, the sellers' cartel solves $\max_\sigma \Pi_S(\beta,\sigma)$ which gives $\sigma(\beta)$, the best reply of the sellers' cartel to any level of market demand β.

A *Nash Equilibrium* of the quantity-setting game is a pair (β^*, σ^*) such that cartels choose simultaneously the quantities to be traded so that each cartel's quantity is a best response to the quantity set by other cartel, namely such that the conditions $\sigma^* \in \sigma(\beta^*)$ and $\beta^* \in \beta(\sigma^*)$ hold.

In what follows, we will sometimes use the notation $\Pi_B^{\mu_B}(\beta,\sigma)$ and $\Pi_S^{\mu_S}(\beta,\sigma)$ to make clear that profits to the buyers' (sellers') cartel depend only on the cartel's own size but not on the other cartel's membership level. Similarly, the best response

[20] It is assumed that each cartel can enforce the exclusion of traders, i.e. its members cannot sneak in the market when they have been ordered to stay out and they cannot organize parallel trade of the excluded quantities.

correspondences will be denoted by $\beta^{\mu_B}(\sigma)$ and $\sigma^{\mu_S}(\beta)$ to stress that, for a given quantity traded by the opponent, the best response of the buyers' (sellers') cartel varies when the cartel's own size varies.[21]

3.1 Market equilibria

At this stage, we will explore the properties and the existence of Nash equilibria (NE) of the quantity-setting game, with given measures of cartel memberships.

The following assertions about the properties of the buyers' cartel payoffs $\Pi_B(\beta, \sigma)$ are useful to gain some intuition about the results. The proofs are straightforward and will be omitted for the sake of brevity. Similar claims hold for $\Pi_S(\beta, \sigma)$.

Claim 1 *The payoff function* $\Pi_B(\beta, \sigma)$ *is continuous at all* (β, σ), *since* $\pi^L(x, x) = \pi^S(x, x)$ *for all* (x, x).

Claim 2 *When* $\sigma > 0$, *the payoff function* $\Pi_B(\beta, \sigma)$ *is strictly increasing in* β *at* $\beta = (1 - \mu_B)\, b$. *Therefore, when supply is positive, the buyers' cartel has always an incentive to let some (i.e. a positive measure) of its members trade.*

Claim 3 *When* $(1 - \mu_B)\, b \leq \beta < \sigma$, *the payoff function* $\Pi_B(\beta, \sigma)$ *is strictly concave in* β *and has at most one critical point in the relevant domain, which is a maximum. Therefore, if* $\Pi_B(\beta, \sigma) \geq \Pi_B(\beta', \sigma)$ *for all* β' *satisfying* $(1 - \mu_B)\, b \leq \beta' < \sigma$, *then* β *is such that* $\beta = \widehat{\beta}(\sigma)$ *where* $\widehat{\beta}(\sigma)$ *solves*

$$\frac{\partial}{\partial \beta}\Pi_B(\beta, \sigma) \equiv \frac{\partial}{\partial \beta}(\beta - (1 - \mu_B)\, b)\, \pi_B^S(\beta, \sigma) = 0. \tag{5}$$

Claim 4 *When* $\sigma \leq \beta \leq b$, *any critical point of the payoff function* $\Pi_B(\beta, \sigma)$ *is a minimum. Therefore, if* $\Pi_B(\beta, \sigma) \geq \Pi_B(\beta', \sigma)$ *for all* $\sigma \leq \beta' \leq b$, *then either* $\beta = b$ *or* $\beta = \sigma$.

Figure 1 plots the payoffs of the buyers' cartel $\Pi_B(\beta, \sigma)$ as a function of market demand β for $b = 1$, $\sigma = \frac{3}{5}$, $\mu_B = \frac{4}{5}$ and various levels of market frictions.

Taking into account the above observations, one can conclude that $\beta(\sigma)$, the best reply of the buyers' cartel to any level of market supply, is one of the following three: (i) to be *inactive*, that is not to withdraw any members and set $\beta(\sigma) = b$, like in Fig. 1a; (ii) to *match* the opponents' quantity, that is to set demand exactly equal to supply $\beta(\sigma) = \sigma$, as in Fig. 1b , or (iii) to *undercut* the quantity traded on the sellers' side and to set demand below supply at a level satisfying condition (5), i.e. $\beta(\sigma) = \widehat{\beta}(\sigma) < \sigma$, as Fig. 1c shows. When does each one of the above choices prevail?[22]

[21] In other words, changes in σ determine movements along the curve $\beta(\sigma)$ whereas changes in μ_B determine a movement of the whole curve $\beta(\sigma)$ and, analogously, changes in β determine movements along the curve $\sigma(\beta)$ whereas changes in μ_S determine a movement of the whole curve $\sigma(\beta)$. See Figs. 2, 3, 4 and Sect. 4 for further reference.

[22] In the sequel, the indeces I, M and U will refer to, respectively, a cartel being inactive, matching the quantity traded on the other side of the market, or undercutting it.

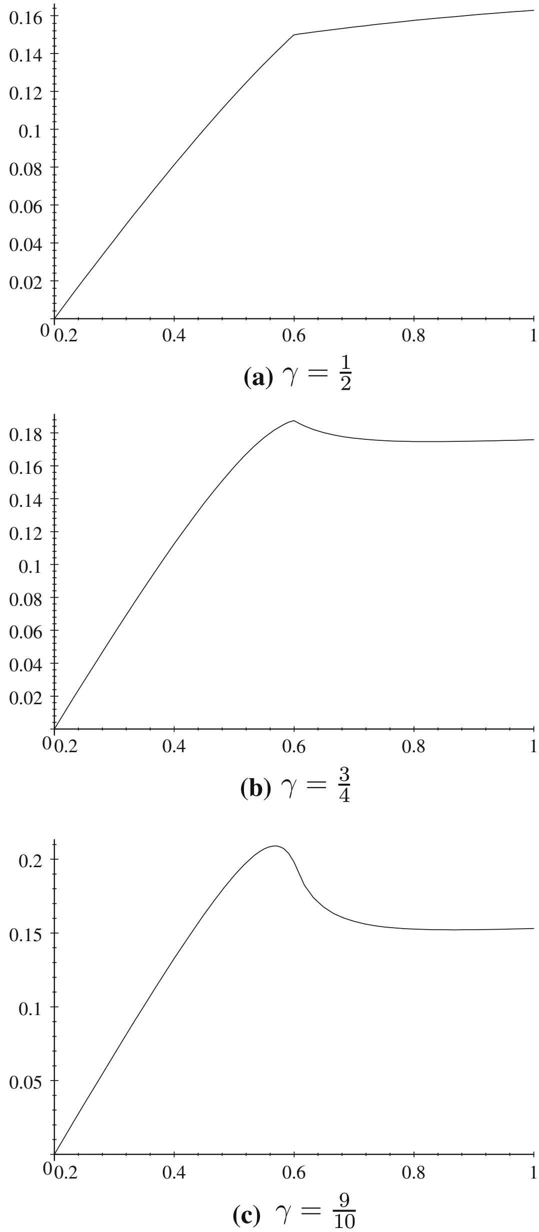

Fig. 1 The buyers' cartel payoff function

Suppose first that undercutting is not a payoff-maximizing solution and that the cartel's decision is based on the comparison between its payoffs when being inactive and when matching the sellers' offer, which are

$$\Pi_B(b,\sigma) = b\mu_B\pi_B^L(b,\sigma) \text{ and } \Pi_B(\sigma,\sigma) = (\sigma - (1-\mu_B)\,b)\,\pi_B^L(\sigma,\sigma),$$

respectively. Denote by $\sigma_{I\sim M}$ the supply level that satisfies $\Pi_B(b,\sigma) = \Pi_B(\sigma,\sigma)$, whereby the buyers' cartel is indifferent between being inactive and matching total supply at $\sigma = \sigma_{I\sim M}$. Then, $\Pi_B(b,\sigma) > \Pi_B(\sigma,\sigma)$ and the buyers' cartel strictly prefers to remain inactive playing $\beta(\sigma) = b$, if

$$\sigma < \frac{2(2-\gamma)(1-\mu_B)b}{\gamma(1+(1-\mu_B)(3-2\gamma))} \equiv \sigma_{I\sim M};$$

or else $\Pi_B(b,\sigma) < \Pi_B(\sigma,\sigma)$ and the buyers' cartel strictly prefers to match the opponent $\beta(\sigma) = \sigma$, if $\sigma > \sigma_{I\sim M}$; whereas, for $\sigma = \sigma_{I\sim M}$, the cartel's best response contains exactly two points and $\beta(\sigma) = \{\sigma; b\}$. Notice that, since $\sigma_{I\sim M}$ is decreasing in the buyers' cartel membership μ_B, the inequality $\Pi_B(b,\sigma) > \Pi_B(\sigma,\sigma)$ is always satisfied when $\sigma_{I\sim M} > s$ or equivalently when

$$\mu_B < \frac{2(2-\gamma)\left(1-\gamma\frac{s}{b}\right)}{2(2-\gamma)-\gamma\frac{s}{b}(3-2\gamma)} \equiv \mu_B^{I\sim M} < 1.$$

Hence, the buyers' cartel always strictly prefers to remain inactive rather than to match total supply if its membership level is low enough, i.e. if $\mu_B < \mu_B^{I\sim M}$.

Let us now address the case in which undercutting could be the payoff-maximizing decision. Observe that a solution to (5) satisfying $\beta < \sigma$ cannot exist when γ is too small, namely when search frictions are sufficiently high. Indeed, condition (5) yields as the unique root in the relevant domain

$$\widehat{\beta}(\sigma) \equiv \frac{\sigma(5-2\gamma) - \sqrt{\sigma(1-\gamma)(5-2\gamma)(\sigma-\gamma(1-\mu_B)b)}}{\gamma(5-2\gamma)},$$

but $\widehat{\beta}(\sigma) < \sigma$ is true only when

$$\gamma > \frac{7-\sqrt{17}}{4} = 0.71922 \equiv \underline{\gamma}. \tag{6}$$

For further reference, we will say that *search frictions are low* when condition (6) is met, otherwise we will say that *search frictions are high*. When search frictions are high, it is not profitable for the buyers' cartel to undercut the sellers' offer σ because the benefits from decreased participation are outweighed by the losses incurred by the "few" active buyers who hardly find a trading partner.[23] For low frictions instead, i.e. when $\gamma > \underline{\gamma}$, the cartel's decision depends on the comparison between its payoffs when undercutting and when matching the sellers' offer, which are

$$\Pi_B(\widehat{\beta}(\sigma),\sigma) = \left(\widehat{\beta}(\sigma) - (1-\mu_B)b\right)\pi_B^S(\widehat{\beta}(\sigma),\sigma) \text{ and}$$
$$\Pi_B(\sigma,\sigma) = (\sigma-(1-\mu_B)b)\,\pi_B^L(\sigma,\sigma),$$

[23] Observe that the same considerations apply to a high discount factor δ in Bloch and Ghosal (2000).

respectively. The inequality $\Pi_B(\widehat{\beta}(\sigma), \sigma) > \Pi_B(\sigma, \sigma)$, together with $\widehat{\beta}(\sigma) < \sigma$ and $\frac{\partial}{\partial \sigma}\widehat{\beta}(\sigma) > 0$, is satisfied if and only if

$$\sigma > \frac{(1-\mu_B)\gamma b}{(1-(5-2\gamma)(1-\gamma))} \equiv \sigma_{M\sim U}.$$

The expression for $\sigma_{M\sim U}$ is decreasing in μ_B and thus inequality $\Pi_B(\widehat{\beta}(\sigma), \sigma) > \Pi_B(\sigma, \sigma)$ can only hold if $\sigma_{M\sim U} < s$, that is if

$$\frac{\gamma - (1-(5-2\gamma)(1-\gamma))\frac{s}{b}}{\gamma} \equiv \mu_B^{M\sim U} < \mu_B \leq 1.$$

Moreover, $\sigma_{I\sim M} \leq \sigma_{M\sim U}$ is true if and only if

$$0 < \mu_B \leq \frac{2(2-\gamma)\left(\gamma^2-(1-(5-2\gamma)(1-\gamma))\right)}{\gamma^2(3-2\gamma)} \equiv \overline{\mu},$$

a condition which is always satisfied when $\overline{\mu} \geq 1$, that is when search frictions are *not too low*, namely when

$$\underline{\gamma} < \gamma \leq 0.78203 \equiv \overline{\gamma},$$

in which case $\mu_B^{I\sim M} \leq \mu_B^{M\sim U} \leq \overline{\mu}$ holds. When $\gamma > \overline{\gamma}$, search frictions will be called *very low*.

When $\overline{\mu} < \mu_B \leq 1$ (implying that market frictions are very low), the payoff-maximizing decision of the buyers' cartel cannot be to match σ, and it is solely based on the comparison between its payoffs from being inactive and from undercutting the quantity offered. Let $\sigma_{I\sim U}$ be defined as the solution to $\Pi_B(b, \sigma) = \Pi_B(\widehat{\beta}(\sigma), \sigma)$, where $\sigma_{M\sim U} < \sigma_{I\sim U} < \sigma_{I\sim M}$ always holds for $\overline{\mu} < \mu_B \leq 1$.[24] Moreover, define $\mu_B^{I\sim U}$ as the solution to $\sigma_{I\sim U} = s$, with $\sigma_{I\sim U} < s$ if and only if $\mu_B > \mu_B^{I\sim U}$. Then, reducing demand to $\widehat{\beta}(\sigma)$ is the cartel's optimal choice when sellers' supply is such that $\sigma > \sigma_{I\sim U}$, whereas remaining inactive is the cartel's payoff-maximizing solution for $\sigma < \sigma_{I\sim U}$, and finally $\beta(\sigma) = \left\{\widehat{\beta}(\sigma); b\right\}$ when $\sigma = \sigma_{I\sim U}$.

The optimal decisions of the sellers' cartel are characterized analogously, with $\beta_{I\sim M}, \beta_{M\sim U}, \beta_{I\sim U}$ defined symmetrically.

For the remainder of the paper, and without loss of generality, we maintain the assumption that sellers are initially the short side of the market and we normalize the measure of the long side.

Assumption 1 Let $s \leq b = 1$.

Observe that, when $s \leq b = 1$, the natural range for the cut-off values $\beta_{I\sim M}, \beta_{M\sim U}$ is not $[0, 1]$ but rather $[0, s]$, because the sellers' cartel cannot match or slightly undercut demand when demand is already above the maximal possible supply level s. Therefore, the critical measures $\mu_S^{I\sim M}$ and $\mu_S^{M\sim U}$ are defined as those membership levels

24 We omit the analytical expression for $\sigma_{I\sim U}$, as it is uninformatively complicated, being $\sigma_{I\sim U}$ one of the roots of a fourth-degree polynomial equation.

satisfying $\beta_{I\sim M} = s$ and $\beta_{M\sim U} = s$, respectively, with $\beta_{I\sim M} < s$ and $\beta_{M\sim U} < s$ if and only if $\mu_S > \mu_S^{I\sim M}$ and $\mu_S > \mu_S^{M\sim U}$, respectively. Conversely, $\mu_S^{I\sim U}$ is the solution to $\beta_{I\sim U} = 1$, with $\beta_{I\sim U} < 1$ if and only if $\mu_S > \mu_S^{I\sim U}$.

This completes the proof of Lemma 1 that follows.

Lemma 1 (a) *When search frictions are high, i.e.* $\gamma \leq \underline{\gamma}$, *the best reply correspondence of the buyers' cartel is*

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \sigma_{I\sim M} \\ \{1, \sigma\} & \text{if } \sigma = \sigma_{I\sim M} \\ \sigma & \text{if } \sigma_{I\sim M} < \sigma \leq s \end{cases} \quad \text{when } \sigma_{I\sim M} < s; \text{ or}$$

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \sigma_{I\sim M} \\ \{1, \sigma\} & \text{if } \sigma = \sigma_{I\sim M} \end{cases} \quad \text{when } \sigma_{I\sim M} = s; \text{ or} \qquad (a)$$

$$\beta(\sigma) = 1 \quad \text{when } \sigma_{I\sim M} > s.$$

Similarly, the best reply of the sellers' cartel is

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \beta_{I\sim M} \\ \{s, \beta\} & \text{if } \beta = \beta_{I\sim M} \\ \min\{\beta, s\} & \text{if } \beta_{I\sim M} < \beta \leq 1 \end{cases} \quad \text{when } \beta_{I\sim M} < s; \text{ or}$$

$$\sigma(\beta) = s \quad \text{when } \beta_{I\sim M} \geq s.$$

(b) *When search frictions are low, i.e.* $\gamma > \underline{\gamma}$, *and when* $\mu_B \leq \overline{\mu}$, *the best reply correspondence of the buyers' cartel is*

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \sigma_{I\sim M} \\ \{1, \sigma\} & \text{if } \sigma = \sigma_{I\sim M} \\ \sigma & \text{if } \sigma_{I\sim M} < \sigma \leq \sigma_{M\sim U} \\ \widehat{\beta}(\sigma) & \text{if } \sigma_{M\sim U} < \sigma \leq s \end{cases} \quad \text{when } \sigma_{I\sim M} < \sigma_{M\sim U} < s; \qquad (b)$$

or like in point (a) *when* $\sigma_{M\sim U} \geq s$. *Similarly (for* $\gamma > \underline{\gamma}$ *and* $\mu_S \leq \overline{\mu}$*), the sellers' cartel best reply correspondence is*

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \beta_{I\sim M} \\ \{s, \beta\} & \text{if } \beta = \beta_{I\sim M} \\ \beta & \text{if } \beta_{I\sim M} < \beta \leq \beta_{M\sim U} \\ \min\{\widehat{\sigma}(\beta), s\} & \text{if } \beta_{M\sim U} < \beta \leq 1 \end{cases} \quad \text{when } \beta_{I\sim M} < \beta_{M\sim U} < s;$$

or like in point (a) *when* $\beta_{M\sim U} \geq s$.

Note that $\widehat{\sigma}(\beta) = \frac{\beta(5-2\gamma) - \sqrt{\beta(1-\gamma)(5-2\gamma)(\beta - \gamma(1-\mu_S)s)}}{\gamma(5-2\gamma)}$.

(*c*) *When market frictions are very low, i.e.* $\gamma > \overline{\gamma}$, *and when* $\overline{\mu} < \mu_B \leq 1$, *the best reply correspondence of the buyers' cartel consists in*

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \sigma_{I\sim U} \\ \{1, \widehat{\beta}(\sigma)\} & \text{if } \sigma = \sigma_{I\sim U} \\ \widehat{\beta}(\sigma) & \text{if } \sigma_{I\sim U} < \sigma \leq s \end{cases} \quad \text{when } \sigma_{I\sim U} < s; \text{ or}$$

$$\beta(\sigma) = \begin{cases} 1 & \text{if } \sigma < \sigma_{I\sim U} \\ \{1, \widehat{\beta}(\sigma)\} & \text{if } \sigma = \sigma_{I\sim U} \end{cases} \quad \text{when } \sigma_{I\sim U} = s; \text{ or} \qquad (c)$$

$$\beta(\sigma) = 1 \text{ when } \sigma_{I\sim U} > s.$$

Similarly (when $\overline{\mu} < \mu_S \leq 1$,*), the sellers' cartel best reply correspondence is*

$$\sigma(\beta) = \begin{cases} s & \text{if } \beta < \beta_{I\sim U} \\ \{s, \widehat{\sigma}(\beta)\}, \text{ provided that } \widehat{\sigma}(\beta) < s & \text{if } \beta = \beta_{I\sim U} \\ \min\{\widehat{\sigma}(\beta), s\} & \text{if } \beta_{I\sim U} < \beta \leq 1 \end{cases} \quad \text{when } \beta_{I\sim U} < 1$$

or $\sigma(\beta) = s$ *when* $\beta_{I\sim U} \geq 1$.[25]

We will say that a cartel is *moderate* when, irrespective of its size, the only relevant options that it faces are either to be inactive or to match the opponent's quantity. A cartel is moderate if and only if search frictions are high, whereby either both the buyers' and the sellers' cartels are moderate or none is. Observe that a moderate cartel will always be inactive if the fraction of its members is such that $\mu_i < \mu_i^{I\sim M}$, with $i = B, S$.

A cartel will be called *radical* if its best reply might consist in either being inactive or undercutting the opponent's quantity (but not matching). A cartel is radical if and only if its size is high enough, i.e. $\overline{\mu} < \mu_i \leq 1$, with $i = B, S$ (a condition implying that market frictions are very low). A radical cartel will always prefer to be inactive if its membership is such that $\overline{\mu} < \mu_i < \mu_i^{I\sim U}$, with $i = B, S$.

Finally, we will say that a cartel is *flexible* if it can potentially respond to the opponent's quantity by staying inactive, matching, or undercutting. A cartel is always flexible if search frictions are *not too low* and it is flexible if and only if search frictions are low and $\mu_i \leq \overline{\mu}$. Nevertheless, a flexible cartel will always remain inactive if $\mu_i < \mu_i^{I\sim M}$, and it will always prefer to match rather than undercut the opponent's volume of trade if $\mu_i^{I\sim M} \leq \mu_i \leq \mu_i^{M\sim U}$. In these cases, the flexible cartel actually behaves as a moderate one.

The buyers' cartel reaction correspondences under the different scenarios contemplated in Lemma 1 are presented and displayed in what follows.

Case 1 Suppose that the buyers' cartel is moderate (i.e. search frictions are high) and $\sigma_{I\sim M} \leq s$. Then its reaction correspondence is as the one displayed in Fig. 2.

[25] Observe that the inequality $\widehat{\sigma}(\beta) < s$ can hold even when $\beta \geq \beta_{I\sim U} > s$. In the case in which $1 > \beta \geq \beta_{I\sim U} > s$ and $\widehat{\sigma}(\beta) \geq s$, the best reply reduces to $\sigma(\beta) = s$.

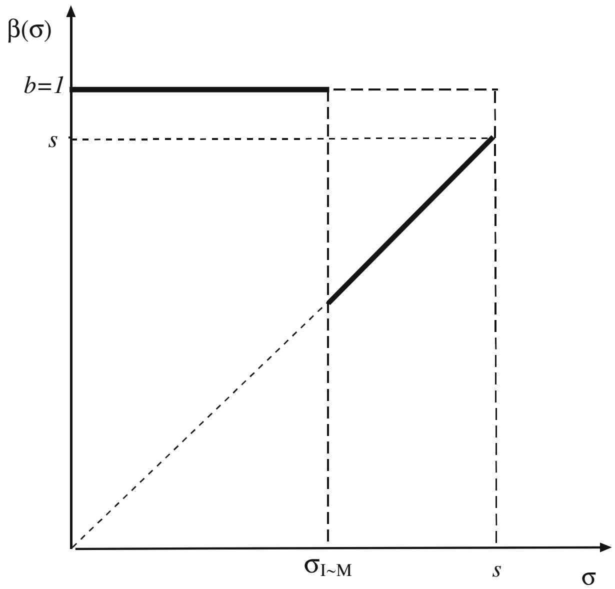

Fig. 2 The best reply of a moderate buyers' cartel

Note that $\sigma_{I\sim M} > s$ for any cartel membership such that $\mu_B < \mu_B^{I\sim M}$; in this case, matching σ is not a payoff-maximizing decision and the buyers' cartel reaction function consists in $\beta(\sigma) = 1$ for all σ.

Case 2 Consider now a flexible buyers' cartel and suppose that $\sigma_{I\sim M} < \sigma_{M\sim U} \leq s$, in which case the buyers' cartel reaction correspondence is represented as in Fig. 3. In the event that $\sigma_{I\sim M} \leq s < \sigma_{M\sim U}$, or else that $\mu_B^{I\sim M} \leq \mu_B < \mu_B^{M\sim U}$, undercutting would not be payoff-maximizing and the buyers' cartel best reply correspondence would look like the moderate cartel's, displayed in Fig. 2. And if $\mu_B < \mu_B^{I\sim M}$ and $\sigma_{I\sim M} > s$, then the buyers' best response is $\beta(\sigma) = 1$ for all σ.

Case 3 Finally consider a radical buyers' cartel. Its reaction correspondence can be represented as in Fig. 4 as long as $\mu_B \geq \mu_B^{I\sim U}$ or $\sigma_{I\sim M} \leq s$ hold. Otherwise, when $\sigma_{I\sim M} > s$ and $\mu_B < \mu_B^{I\sim U}$, $\beta(\sigma) = 1$ for all σ.

The best response correspondences characterized in Lemma 1 generate two distinct types of Nash equilibria of the quantity-setting game.

On the one hand, the best responses may overlap for a non-empty interval along the diagonal. In this case, all Nash equilibria yield a perfect match in the measures of active buyers and sellers, and at least one cartel actively restrains trade. This symmetric trade scenario is attained when search frictions are high and either $\sigma_{I\sim M} < s$ and $\beta_{I\sim M} < s$ both hold (in which case both cartels restrain trade), or $\sigma_{I\sim M} = s$ and no restriction is placed on $\beta_{I\sim M}$ (a case in which the buyers' cartel matches s). When search frictions are low, an equilibrium with symmetric participation can prevail only if both cartels are flexible (i.e. if both $\sigma_{I\sim M} \leq \sigma_{M\sim U}$ and $\beta_{I\sim M} \leq \beta_{M\sim U}$ hold, a constraint binding only under not too low frictions), and if and only if the best response

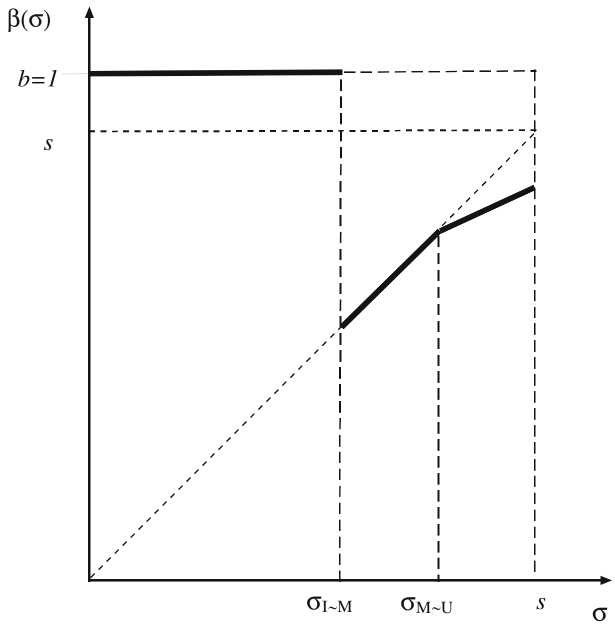

Fig. 3 The best reply of a flexible buyers' cartel

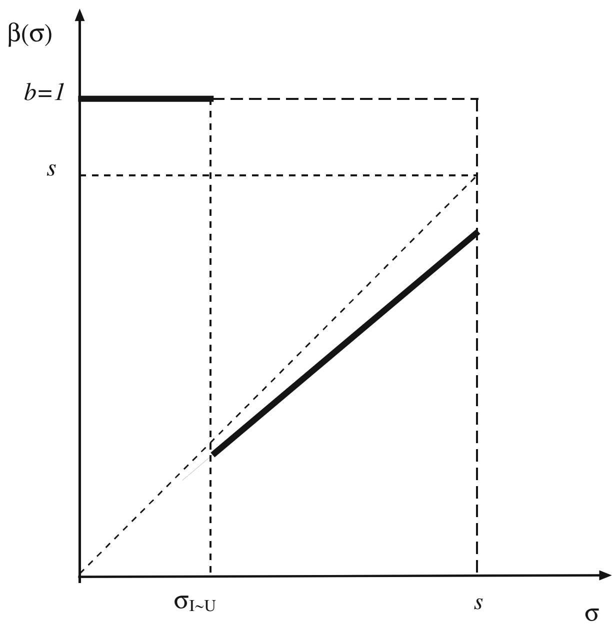

Fig. 4 The best reply of a radical buyers' cartel

correspondences overlap along the diagonal, that is

$$\underline{q} \equiv \min \{\max \{\beta_{I\sim M}, \sigma_{I\sim M}\}, s\} \leq \min \{\beta_{M\sim U}, \sigma_{M\sim U}, s\} \equiv \overline{q}. \quad (7)$$

Observe that the following relationships always hold among cut-off values: if $\beta_{I\sim M} \geq \sigma_{I\sim M}$ then $\beta_{M\sim U} > \sigma_{M\sim U}$ and, conversely, if $\sigma_{M\sim U} \geq \beta_{M\sim U}$ then $\sigma_{I\sim M} > \beta_{I\sim M}$.

On the other hand, when the best response correspondences do not overlap along the diagonal, they may intersect at the boundaries. This yields a Nash equilibrium with asymmetric participation where one cartel is inactive and the other cartel restrains its market participation so that it slightly undercuts the opponent's. Alternatively, there exist equilibria with symmetric participation, such that the sellers' cartel is inactive and the buyers' cartel matches s, or equilibria where both cartels remain inactive.[26]

The relative magnitude of cartel memberships, μ_B and μ_S, determines whether a Nash equilibrium of the quantity-setting game exists, and, if so, whether it is a symmetric NE or an asymmetric one. In particular, if membership levels are close to each other and sufficiently high, then condition (7) holds and symmetric equilibria, with at least one cartel restraining its market participation, are attained. Conversely, if cartel memberships are sufficiently different, a unique Nash equilibrium prevails in which the quantities traded are asymmetric, and where only the larger cartel restrains trade. Finally, for intermediate cases, the quantity-setting game might not have a Nash equilibrium.

The proposition below characterizes the Nash equilibria of the quantity-setting game, and gives the necessary and sufficient conditions for their existence. The proof follows straightforwardly from inspection of the best response correspondences.[27]

Proposition 1 (*a*) *Let search frictions be high (i.e. $\gamma \leq \underline{\gamma}$, implying that both cartels are moderate). If being inactive and matching are both payoff-maximizing options for the two cartels, i.e. if both $\sigma_{I\sim M} \leq s$ and $\beta_{I\sim M} < s$ hold (equivalently if $\mu_B \geq \mu_B^{I\sim M}$ and $\mu_S > \mu_S^{I\sim M}$), then any strategy pair $(\beta, \sigma) = (q, q)$, with $q \in \left[\underline{q}, s\right]$ and $\underline{q} \equiv \min \{\max \{\beta_{I\sim M}, \sigma_{I\sim M}\}, s\}$ is a NE. Otherwise, when $\sigma_{I\sim M} > s$, the strategy pair $(\beta, \sigma) = (1, s)$ is the unique NE.*

(*b*) *Let search frictions be low, i.e. $\gamma > \underline{\gamma}$. Then, a strategy pair $(\beta, \sigma) = (q, q)$, with $q \in \left[\underline{q}, \overline{q}\right]$, is a NE if and only if both cartels are flexible, i.e. $\mu_B, \mu_S \leq \overline{\mu}$, and (7) is satisfied (meaning that both $\sigma_{I\sim M} \leq s$ and $\beta_{I\sim M} < s$ hold). Otherwise, the unique NE is*

26 For the sake of completeness, let us point out that, when $\sigma_{I\sim M} = s$, there exist values for β such that both equilibria (s, s) and $(1, s)$ coexist. Similarly, when $\sigma_{I\sim U} = s$, there exist values for β such that both equilibria $\left(\widehat{\beta}(s), s\right)$ and $(1, s)$ coexist. Thus, when best replies intersect at the boundaries, a Nash equilibrium is not always unique.

27 See Example 1 below.

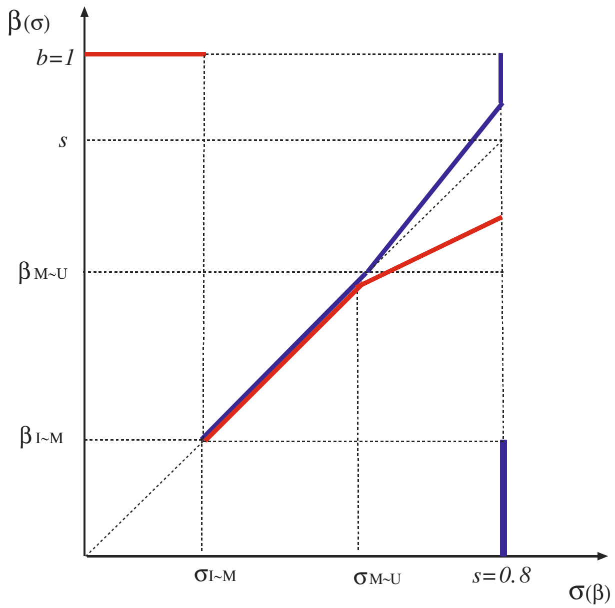

Fig. 5 Multiplicity of equilibria

$$
(\beta,\sigma)=\begin{cases}\left(\widehat{\beta}(s),s\right) & \textit{if } \widehat{\beta}(s)\le\min\{\beta_{I\sim M},\beta_{I\sim U}\} \textit{ and if } \widehat{\beta}(s)<s \textit{ or} \\ & \quad \max\{\sigma_{M\sim U},\sigma_{I\sim U}\}<s \\ (1,\widehat{\sigma}(1)) & \textit{if } \widehat{\sigma}(1)\le\min\{\sigma_{I\sim M},\sigma_{I\sim U}\} \textit{ and if } \widehat{\sigma}(1)<s,\ \textit{being} \\ & \quad \beta_{M\sim U}<s \textit{ or } \beta_{I\sim U}<1 \\ (1,s) & \textit{if } s\le\min\{\sigma_{I\sim M},\sigma_{I\sim U}\} \textit{ and either } \beta_{M\sim U}<s \textit{ or} \\ & \quad \beta_{I\sim U}<1 \textit{ but } \widehat{\sigma}(1)\ge s;\ \textit{or else if } s\le\min\{\sigma_{I\sim M},\sigma_{I\sim U}\} \\ & \quad \textit{and either } \beta_{M\sim U}\ge s \textit{ or } \beta_{I\sim U}\ge 1.\end{cases}; \tag{8}
$$

No NE exists when the conditions in (8) are not met.

Example 1 considers the range of equilibrium outcomes for flexible cartels.

Example 1 Let $s=\frac{4}{5}$ be the ex ante measure of sellers, let search frictions be equal to $\gamma=\frac{3}{4}$ and let the proportion of buyers in the cartel be $\mu_B=\frac{20}{21}$.

1. If the proportion of sellers in the cartel is $\mu_S=\frac{31}{33}$ then the cartel game is such that all pairs $(\beta,\sigma)=(q,q)$ with $q\in\left[\frac{4}{27},\frac{2}{7}\right]=[0.148, 0.286]$ represent equilibrium outcomes. This result is shown in Fig. 5.
2. Assume that $\mu_S=\frac{23}{50}$, then the unique equilibrium of the quantity-setting game is given by the pair $\left(\widehat{\beta}(s),s\right)=\left(\frac{112-2\sqrt{214}}{105},\frac{4}{5}\right)=(0.788, 0.8)$ and is shown in Fig. 6.

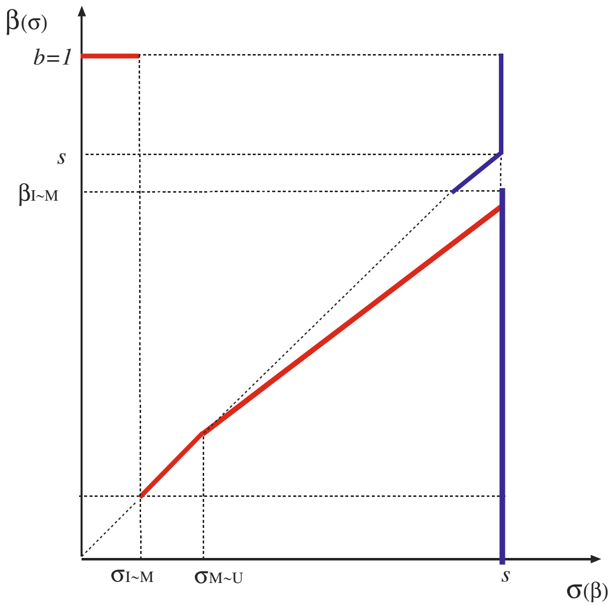

Fig. 6 Uniqueness of the equilibrium

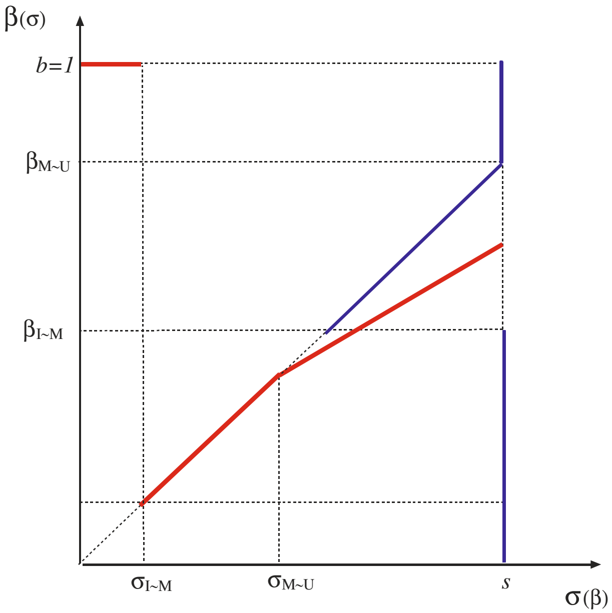

Fig. 7 Non-existence of an equilibrium

3. Finally, letting $\mu_S = \frac{5}{6}$, it is straightforward to check that the quantity-setting game has no Nash equilibrium, as shown in Fig. 7.

4 Stability

Depending on the magnitudes of cartel memberships μ_B and μ_S, a broad set of Nash equilibria of the quantity-setting game might exist. But not all equilibria are equally relevant, because some of them cannot be supported by stable levels of cartel memberships.

Indeed, an equilibrium in the quantity-setting game is a reasonable prediction for the operation of the market only when cartels can be expected to maintain their membership levels. Cartels might not preserve their sizes, either because some of their members wish to defect and become free traders, or because some free traders wish to join the cartel.

With a continuum of non-atomic agents, the defection of a single agent has a negligible impact on the final outcome, therefore the notion of stability must not rely directly on immunity from unilateral deviations. Our concept of cartel stability first postulates the absence of incentives to deviate by coalitions of small but strictly positive measure (this is what is called ε-stability). Then, we study what happens when one lets the measure of the deviating coalitions go to zero (this will be called stability).

Let a *market outcome* be a profile $(\mu_B, \mu_S, \beta, \sigma)$ such that (β, σ) is a Nash equilibrium of the quantity-setting game when the (now endogenous) cartel memberships are (μ_B, μ_S). Given the market outcome $(\mu_B, \mu_S, \beta, \sigma)$, we will say that an *$\varepsilon$-coalition* of buyers in the cartel (that is a measure $0 < \varepsilon < \mu_B$ of cartel members) *benefits from defecting the cartel* if and only if the individual payoffs to each buyer in the deviating coalition are such that

$$\pi_B(\beta^{\mu_B-\varepsilon}(\sigma), \sigma) > \frac{\Pi_B(\beta, \sigma)}{\mu_B}. \tag{9}$$

Similarly, an *ε-coalition* of buyers that are outsiders to the cartel (that is a measure $0 < \varepsilon < (1 - \mu_B)$ of non-members to the cartel) *benefits from joining the cartel* if and only if

$$\frac{\Pi_B\left(\beta^{\mu_B+\varepsilon}(\sigma), \sigma\right)}{(\mu_B + \epsilon)} > \pi_B(\beta, \sigma). \tag{10}$$

The conditions under which an ε-coalition of sellers benefits from a deviation can be expressed analogously, the only difference being that cartel members can form a deviating coalition of size $\varepsilon \in (0, \mu_S s)$ while outsiders can form a deviating coalition of size $\varepsilon \in (0, (1 - \mu_S) s)$.

Notice that, in assessing whether an ε-coalitional deviation is profitable or not at the market outcome $(\mu_B, \mu_S, \beta, \sigma)$, the quantity traded and the cartel's membership on the other side of the market are taken as given. Moreover, it is assumed that a cartel can perfectly observe if it is affected by a deviation, and the change in the cartel's size will in turn alter the cartel's optimal response to the quantity set by the counterpart.

A cartel is *ε-stable* at the profile $(\mu_B, \mu_S, \beta, \sigma)$ if there exists an $\varepsilon > 0$ such that no coalitions of its members (non-members), whose size is *at least* as great as ε, benefit from defecting (joining) the cartel. A profile $(\mu_B, \mu_S, \beta, \sigma)$ is an *ε-stable* *market outcome* if and only if: (i) it is a Nash equilibrium of the quantity-setting game, and (ii) both cartels are ε-stable.

A cartel is *stable* at the profile $(\mu_B, \mu_S, \beta, \sigma)$ if for each $\delta > 0$ there exists a measure $\varepsilon \leq \delta$ of agent such that no coalitions of size ε of its members (non-members) benefit from defecting (joining) the cartel. A profile $(\mu_B, \mu_S, \beta, \sigma)$ is a *stable market outcome* if and only if: (i) it is a Nash equilibrium of the quantity-setting game, and (ii) both cartels are stable.

Observe that, in the present model, cartels generate positive externalities that benefit outsiders on the same side of the market. Thus, the incentives of insiders to defect from the cartel may strongly undermine the stability of an equilibrium of the quantity-setting game. Conversely, the incentives of outsiders to join the cartel will not be a concern. In other words, condition (9) is the one that will matter, while condition (10) will never have a bite.

We will show that, for a wide set of parameter configurations, there exist ε-stable (and stable) market outcomes with at least one active cartel. At these market outcomes, the quantities traded must be symmetric, regardless of the potential measures of traders on each side of the market. If frictions are low, ε-stable (and stable) market outcomes, where only one cartel is active, exist as well. At these outcomes, the active cartel is the one with greater absolute membership and it reduces its supply or demand so as to slightly undercut the counterpart's (unconstrained) market participation.

Our next result establishes necessary and sufficient conditions for ε-stability.

Proposition 2 (*i*) *A market outcome such that trade is symmetric and at least the buyers' cartel is active, i.e. a profile* (μ_B, μ_S, q, q)*, is* ε*-stable if and only if either* (*a*) *neither cartel is radical, being* $\mu_B, \mu_S \leq \overline{\mu}$*, trade is set at the level* $q < s$ *and deviating buyers and sellers in their cartels have measures* $\varepsilon_B^* < \varepsilon_B < \mu_B - \mu_B^{I\sim M}$ *and* $\varepsilon_S^* < \varepsilon_S < \mu_B - \mu_S^{I\sim M}$ *respectively; or* (*b*) *neither cartel is radical, being* $\mu_B, \mu_S \leq \overline{\mu}$*, and trade is set at* (s, s) *with* $\sigma_{I\sim M} = s$*, that is* $\mu_B = \mu_B^{I\sim M}$*, and with the sellers' cartel preferring to be inactive, being* μ_S *any measure for which* $s \in \sigma(s)$*.*

(*ii*) *A market outcome such that trade is asymmetric and only the buyers' cartel is active, i.e. a profile* $\left(\mu_B, \mu_S, \widehat{\beta}(s), s\right)$*, is* ε*-stable if and only if search frictions are low, the sellers' cartel prefers to be is inactive, i.e.* $\mu_S \leq \min\left\{\mu_S^{I\sim U}, \mu_S^{M\sim U}\right\}$*, and either the buyers' cartel is flexible,* $s > \max\{s^*, s^{**}\}$*,* $\mu_B^* \leq \mu_B \leq \overline{\mu}$ *and* $\varepsilon_B \geq \mu_B - \mu_B^{M\sim U}$*, or the buyers' cartel is radical and* $\mu_B - \mu_B^{I\sim U} < \varepsilon_B < \mu_B - \overline{\mu}$*.*

(*iii*) *A market outcome such that trade is asymmetric and only the sellers' cartel is active, i.e. a profile* $(\mu_B, \mu_S, 1, \widehat{\sigma}(1))$ *with* $\widehat{\sigma}(1) < s$*, is* ε*-stable if and only if search frictions are low, the buyers' cartel always prefers to be inactive, i.e.* $\mu_B \leq \min\left\{\mu_B^{I\sim M}, \mu_B^{I\sim U}\right\}$*, and either the sellers' cartel is flexible with* $s > \max\{s_*, s_{**}\}$*,* $\mu_S^* < \mu_S \leq \overline{\mu}$ *and* $\varepsilon_S > \mu_S - \mu_S^*$*, or the sellers' cartel is radical with* $s > s^{**}$ *and* $\mu_S - \mu_S^{I\sim U} < \varepsilon_S < \mu_S - \max\left\{\overline{\mu}, \mu_S^*\right\}$*.*

Proof For the relevant bounds ε_i^*, $\mu_i^* \ldots$, with $i = B, S$, and the Proof, see Appendix A. □

Example 2 Going back to Example 1, where $s = \frac{4}{5}$, $\gamma = \frac{3}{4}$ and $\mu_B = \frac{20}{21}$, one obtains the following results:

1. The profile $(\mu_B, \mu_S, q, q) = \left(\frac{20}{21}, \frac{31}{33}, q, q\right)$ is an ε-stable market outcome if and only if, for any given quantity $q \in \left(\frac{4}{27}, \frac{2}{7}\right)$, the deviating coalitions have measures $\frac{5}{21}\frac{27q-4}{20-9q} < \varepsilon_B < \frac{55}{168} = 0.327$ for the buyers' cartel and $\frac{8}{33}\frac{27q-4}{16-9q} < \varepsilon_S < \frac{16}{33} = 0.485$ for the sellers' cartel.[28] Assuming the quantity set at equilibrium is $q = \frac{1}{5}$, then the market outcome is ε-stable for any $0.0183 = \frac{5}{273} < \varepsilon_B < 0.327$ and any $0.0239 = \frac{56}{2343} < \varepsilon_S < 0.485$.
2. The profile $\left(\mu_B, \mu_S, \widehat{\beta}(s), s\right) = \left(\frac{20}{21}, \frac{23}{50}, 0.78, 0.8\right)$ is not an ε-stable market outcome, irrespective of the size of the deviating coalition ε_B.

Observe that an ε-stable outcome with only the sellers' cartel being active might exist when the market is balanced, i.e. if $s = b = 1$, or only if the total measure of sellers s is very close to the total measure of buyers b. In this case the sellers' cartel is active but only marginally so.

Also note that, among ε-stable market outcomes with asymmetric participation, only those where the fraction of cartel members is $\mu_i = \mu_i^{I\sim U}$, with $i = B, S$, survive when the measure of the deviating coalition ε becomes arbitrarily close to zero.

A characterization of the set of stable market outcomes is presented in the next proposition, whose proof is immediate and therefore omitted.

Proposition 3 (i) *A market outcome such that trade is symmetric and at least the buyers' cartel is active, i.e. a profile (μ_B, μ_S, q, q), is stable if and only if either (a) neither cartel is radical, being $\mu_B, \mu_S \leq \overline{\mu}$, and trade is set at $q = \beta_{I\sim M} = \sigma_{I\sim M} < s$; or (b) it belongs to the set of ε-stable outcomes under case ($i.b$).*

(ii) *A market outcome such that trade is asymmetric and only the buyers' cartel is active, i.e. a profile $\left(\mu_B, \mu_S, \widehat{\beta}(s), s\right)$, is stable if and only if $\mu_B = \mu_B^{I\sim U}$ and μ_S is such that $\widehat{\beta}(s) \leq \min\{\beta_{I\sim M}, \beta_{I\sim U}\}$.*

(iii) *A market outcome such that trade is asymmetric and only the sellers' cartel is active, i.e. a profile $(\mu_B, \mu_S, 1, \widehat{\sigma}(1))$, with $\widehat{\sigma}(1) < s$, is stable if and only if $s > s^{**}$, $\mu_S = \mu_S^{I\sim U} > \mu_S^*$ and μ_B is such that $\widehat{\sigma}(1) \leq \min\{\sigma_{I\sim M}, \sigma_{I\sim U}\}$.*

Remark 1 Note that $q = \beta_{I\sim M} = \sigma_{I\sim M}$ is equivalent to

$$0 < \mu_B = 1 - \frac{(1-\mu_S)\,s}{1 + (1-\mu_S)(3-2\gamma)(1-s)} \leq 1, \tag{11}$$

so that, to every pair of cartel memberships satisfying (11), there corresponds a stable market outcome with symmetric market participation. Moreover, there always exist pairs of cartel memberships meeting the stability requirements for a market outcome with asymmetric trade.

[28] According to Proposition 4 that follows, the profile $\left(\frac{20}{21}, \frac{31}{33}, \frac{4}{27}, \frac{4}{27}\right)$ is the only stable market outcome.

The level of trade at stable market outcomes with symmetric participation might be inefficient. Moreover, condition (11) is compatible with market outcomes in which the cartels control more and more traders and force very limited market participation. In the limit, the market is driven to the collapse, given that the profile (1, 1, 0, 0), is a stable market outcome for any level of market frictions.

Corollary 1 *The set of stable market outcomes with symmetric trade includes inefficient profiles such that* $0 \leq \beta_{I\sim M} = \sigma_{I\sim M} < s$. *The degree of inefficiency is unbounded.*

Nonetheless, Proposition 3 does not rule out market outcomes at which quantities are set equal to the measure of the short side, $\beta = \sigma = s$, thus the following holds.

Corollary 2 *Stable market outcomes include efficient profiles where* $\mu_B = \mu_B^{I\sim M}$ *and* μ_S *is such that* $s \in \sigma(s)$. *It is an outcome with symmetric participation, where the sellers' cartel is not active and the buyers' is active only to assure that demand equals supply.*

At stable market outcomes with symmetric participation and both cartels being active, the cartel on the short side always withdraws from the market fewer traders than the cartel on the long side does. Does the cartel on the long side necessarily control more traders than its opponent? Does it need to encompass a relatively higher proportion of traders? These questions are answered in the following corollary.

Corollary 3 *At stable market outcomes with symmetric trade and both cartels being active:* (i) *the buyers' cartel controls more traders than the sellers' cartel, i.e.* $\mu_B > s\mu_S$; (ii) *the buyers' cartel always controls a larger fraction of the total population on its side than the sellers' cartel, i.e.* $\mu_B > \mu_S$.

Proof Taking into account condition (11), the inequality $\mu_B > s\mu_S$ holds if and only if

$$s\mu_S < 1 - \frac{(1-\mu_S)\,s}{1 + (1-\mu_S)\,(3-2\gamma)\,(1-s)}, \tag{12}$$

and it is straightforward to check that (12) holds for all μ_S and s. Inequality $\mu_S < \mu_B$ also holds for all $s < 1$. □

At stable outcomes with asymmetric trade, the only active cartel slightly undercuts the opponent's participation level. Even though the active cartel actually withdraws a non-negligible measure of its members, efficiency is only marginally affected, since the level of realized trade decreases only by a small amount. The distribution of surplus between buyers and sellers is again substantially altered in favor of the side of the market where the cartel is active.

We may now summarize our results. There are stable market outcomes where both sides exercise countervailing power. In this case, countervailing power might be the source of (potentially severe) market inefficiency. However, in markets that are not balanced *ex ante* (maybe because the short side has effective means to prevent entry without compensation to excluded potential traders) there exist stable outcomes at which the exercise of countervailing power by the long side affects the distribution of surplus without damaging efficiency. We find that these market outcomes (partially) vindicate Galbraith's claims that countervailing power plays a desirable role in some markets.

Appendix A: Proof of proposition 2

It is clear that there are no situations where a positive measure ε of free buyers or sellers have incentive to deviate and to join the cartel on their side of the market. Indeed, outsiders obtain a higher payoff than insiders provided that the cartel is active. Hence, the only relevant deviations are represented by a positive measure ε of cartel members wishing to defect from the cartel.

(a) Consider first market outcomes with symmetric trade.

- These outcomes can arise when the best replies overlap along the diagonal, i.e. for cartel memberships such that $\mu_B, \mu_S \leq \overline{\mu}$ (cartels can be either moderate or flexible). In particular, assume that the fractions of cartel members are such that

$$\max\{\beta_{I\sim M}, \sigma_{I\sim M}\} = \sigma_{I\sim M} \leq \min\{\beta_{M\sim U}, s\} = \min\{\beta_{M\sim U}, \sigma_{M\sim U}, s\}$$

whereby equilibrium strategies are such that $(\beta, \sigma) = (q, q)$, with

$$\sigma_{I\sim M}^{\mu_B} = \frac{2(2-\gamma)(1-\mu_B)}{\gamma(1+(1-\mu_B)(3-2\gamma))} \leq q \leq \min\{\beta_{M\sim U}, s\}.$$

Suppose further that, at a given market outcome, a strictly positive measure ε_B of cartel members leave the buyers' cartel. When such a defection occurs, the best reply correspondence of the buyers' cartel shifts towards the right, due to a decrease in cartel membership from μ_B to $\mu_B - \varepsilon_B$. If the equilibrium measure of active traders q is such that $\sigma_{I\sim M}^{\mu_B-\varepsilon_B} < q \leq \min\{\beta_{M\sim U}, s\}$, then leaving the cartel is beneficial. Indeed, after the defection, the buyers' cartel continues to set its measure of active traders equal to $\beta^{\mu_B-\varepsilon_B}(q) = q$, which yields *per capita* payoffs $\pi_B(q, q)$ to outsiders (and to defecting cartel members). Prior to the defection, the individual payoff to cartel members is $\frac{\Pi_B^{\mu_B}(q,q)}{\mu_B} < \pi_B(q, q)$. However, if q is such that $\sigma_{I\sim M}^{\mu_B} \leq q < \sigma_{I\sim M}^{\mu_B-\varepsilon_B}$, then the buyers' cartel breaks down completely as a consequence of the defection, and it plays $\beta^{\mu_B-\varepsilon_B}(q) = 1$. In this situation, defecting buyers would each receive a payoff equal to $\pi_B^L(1, q)$, and a defection would not be profitable if $\frac{\Pi_B^{\mu_B}(q,q)}{\mu_B} \geq \pi_B^L(1, q)$, which is indeed the case.[29] Therefore, for any given symmetric outcome of the quantity-setting game

[29] The case in which $q = \sigma_{I\sim M}^{\mu_B-\varepsilon_B}$ is debatable, because the best response of the cartel is not unique, being $\beta^{\mu_B-\varepsilon_B}(q) = \{q, 1\}$. When $q = \sigma_{I\sim M}^{\mu_B-\varepsilon_B}$, a deviation might be profitable if the buyers' cartel switches from q to 1.

$q \in \left[\sigma_{I\sim M}^{\mu_B}; \min\{\beta_{M\sim U}, s\}\right]$, one can find a *minimal* coalition size ε_B^*, where ε_B^* is the solution to $\sigma_{I\sim M}^{\mu_B-\varepsilon_B} = q$, such that no coalition of buyers whose measure is $\varepsilon_B \geq \varepsilon_B^*$ finds it profitable to deviate and leave the buyer' cartel. Of course, the deviating coalition should not be excessively big so that $\sigma_{I\sim M}^{\mu_B-\varepsilon_B} \geq s$, hence $\varepsilon_B < \mu_B - \mu_B^{I\sim M}$.
The same reasoning applies when $\max\{\beta_{I\sim M}, \sigma_{I\sim M}\} = \beta_{I\sim M} \leq \min\{\sigma_{M\sim U}, s\} = \min\{\beta_{M\sim U}, \sigma_{M\sim U}, s\}$ or when one considers the sellers' cartel: ε-stability is guaranteed for any $q \in [\max\{\beta_{I\sim M}, \sigma_{I\sim M}\}; \min\{\beta_{M\sim U}, \sigma_{M\sim U}, s\}]$ when deviating sellers have measure $\varepsilon_S > \varepsilon_S^*$, being ε_S^* the solution to $\beta_{I\sim M}^{\mu_S-\varepsilon_S} = q$, with $\varepsilon_S < \mu_S - \mu_S^{I\sim M}$.
Finally note that a market outcome with symmetric trade (μ_B, μ_S, q, q) with $\mu_B^{I\sim M} < \mu_B \leq \overline{\mu}$ and $\mu_S^{I\sim M} < \mu_S \leq \overline{\mu}$ is stable, no matter how small the size $\varepsilon > 0$ of the potentially deviating coalitions are, if and only if

$$q = \beta_{I\sim M} = \frac{2(2-\gamma)(1-\mu_S)s}{\gamma(1+(1-\mu_S)(3-2\gamma))}$$
$$= \frac{2(2-\gamma)(1-\mu_B)}{\gamma(1+(1-\mu_B)(3-2\gamma))} = \sigma_{I\sim M},$$

which implies the following relationship between cartels' memberships

$$\mu_B = 1 - \frac{(1-\mu_S)s}{1+(1-\mu_S)(3-2\gamma)(1-s)}.$$

- Consider now the case in which a market outcome with symmetric trade arises when the reaction correspondences intersect at the boundaries. It is straightforward to check that deviations are not profitable if and only if the sellers' cartel prefers to be inactive and the buyers' cartel just matches s, that is for $\sigma_{I\sim M} = s$, i.e. $\mu_B = \mu_B^{I\sim M}$, and for any μ_S such that $s \in \sigma(s)$.

$(b.1)$ Secondly, consider market outcomes with asymmetric trade and suppose that the strategy pair $\left(\widehat{\beta}(s), s\right)$ is played.

- Suppose, for the time being, that $\max\{\sigma_{M\sim U}, \sigma_{I\sim U}\} = \sigma_{M\sim U}$ (so that the buyers' cartel is flexible and $\mu_B \leq \overline{\mu}$) and consider a potential defection from the buyers' cartel. After the deviation, the measure of buyers in the cartel becomes $\mu_B - \varepsilon_B$ and again the cartel's best reply correspondence shifts slightly towards the right. Such a defection has two possible consequences, depending on the magnitude of ε_B. (i) The buyers' cartel continues to respond to s by setting $\beta(s) = \widehat{\beta}^{\mu_B-\varepsilon_B}(s) < s$. This occurs when $\sigma_{M\sim U}^{\mu_B-\varepsilon_B} < s$, or equivalently when $\varepsilon_B < \mu_B - \mu_B^{M\sim U}$. We claim that, when no cartel is active on the supply side, it is profitable for a measure $\varepsilon_B > 0$ of members of the buyers' cartel to defect. Observe that the *per capita* utility of outsiders after the defection is equal to

$$\pi_B^S\left(\widehat{\beta}^{\mu_B-\varepsilon_B}(s), s\right) = \gamma\frac{(5-2\gamma)(s-\gamma(1-\mu_B+\varepsilon_B))-\sqrt{(s(1-\gamma)(5-2\gamma)(s-\gamma(1-\mu_B+\varepsilon_B)))}}{4(s-\gamma(1-\mu_B+\varepsilon_B))}, \tag{13}$$

whereas the *per capita* utility that cartel members receive prior to the defection is

$$\frac{\Pi_B\left(\widehat{\beta}^{\mu_B}(s), s\right)}{\mu_B} = \frac{(s-(1-\mu_B)\gamma)(5-2\gamma)+s(1-\gamma)-2\sqrt{s(1-\gamma)(5-2\gamma)(s-(1-\mu_B)\gamma)}}{4\mu_B}. \tag{14}$$

Furthermore note that $\pi_B^S\left(\widehat{\beta}^{\mu_B}(s), s\right) > \frac{\Pi_B\left(\widehat{\beta}^{\mu_B}(s), s\right)}{\mu_B}$ always holds and that (13) is decreasing in ε_B. Thus, for ε_B small enough, (13) is strictly greater than (14). (ii) The buyers' cartel sets $\beta(s) = s$. Then, it must be that $\sigma_{M\sim U}^{\mu_B-\varepsilon_B} \geq s$, or equivalently that $\varepsilon_B \geq \mu_B - \mu_B^{M\sim U}$. When this defection occurs, cartel members have individual payoff given by (14) before the defection, which is at least as great as the outsiders' individual payoff, i.e. $\pi_B(s, s) = \frac{1}{2}\gamma(2-\gamma)$, if and only if

$$\mu_B \geq \frac{\left((5-2\gamma)\gamma-\left(-4+11\gamma-4\gamma^2\right)s\right)+\sqrt{8s\gamma(5-2\gamma)(1-\gamma)(2-\gamma)(1-s)}}{\gamma} \equiv \mu_B^* > \mu_B^{M\sim U},$$

where $\mu_B^* < 1$ only when $s > \frac{2\gamma(2-\gamma)\left(3(2-\gamma)+2\sqrt{(1-\gamma)(5-2\gamma)}\right)}{(4-\gamma)^2} \equiv s^*$. But, in order for the buyers' cartel to be flexible, it must be that $\mu_B \leq \overline{\mu}$, which is always the case when $\overline{\mu} \geq 1$ or else $\gamma \leq \overline{\gamma}$. When $\gamma > \overline{\gamma}$, instead, inequality $\mu_B^* < \overline{\mu}$ needs to hold, which happens if and only if

$$s > \frac{\left(-28\gamma^5+202\gamma^4-533\gamma^3+624\gamma^2-320\gamma+64\right)+4(1-\gamma)(2-\gamma)\sqrt{\gamma(5-2\gamma)(4-3\gamma)\left(8\gamma^3-37\gamma^2+48\gamma-16\right)}}{\gamma(3-2\gamma)(4-\gamma)^2} \equiv s^{**},$$

with $s^{**} > s^*$ if and only if $\gamma > \overline{\gamma}$. Therefore, the profile $\left(\mu_B, \mu_S, \widehat{\beta}(s), s\right)$ is ε-stable for any $s > \max\{s^*, s^{**}\}$, any $\mu_B^* \leq \mu_B \leq \overline{\mu}$, any $\varepsilon_B \geq \mu_B - \mu_B^{M\sim U}$, and for any $\mu_S \leq \min\left\{\mu_S^{I\sim M}, \mu_S^{I\sim U}\right\}$.

- When instead $\max\{\sigma_{M\sim U}, \sigma_{I\sim U}\} = \sigma_{I\sim U}$ (in which case the buyers' cartel is radical and $\overline{\mu} < \mu_B \leq 1$), the following cases have to be considered when a defection from the buyers' cartel occurs. (i) If $\sigma_{I\sim U}^{\mu_B-\varepsilon_B} < s$, or equivalently $\mu_B - \varepsilon_B > \mu_B^{I\sim U}$, and the buyers' cartel continues to respond to s setting $\beta(s) = \widehat{\beta}^{\mu_B-\varepsilon_B}(s) < s$, then the deviation is profitable. (ii)

When $\sigma_{I\sim U}^{\mu_B-\varepsilon_B} > s$, or equivalently $\mu_B - \varepsilon_B < \mu_B^{I\sim U}$, the defection from the cartel triggers the response $\beta^{\mu_B-\varepsilon_B}(s) = 1$, in which case the equilibrium outcome is $(1, s)$ and the deviating members are not better off. [30] Then the profile $\left(\mu_B, \mu_S, \widehat{\beta}(s), s\right)$ is an ε-stable market outcome for every $\mu_B - \mu_B^{I\sim U} < \varepsilon_B < \mu_B - \overline{\mu}$ and for every $\mu_S \leq \min\left\{\mu_S^{I\sim M}, \mu_S^{I\sim U}\right\}$.

$(b.2)$ Finally, consider market outcomes of the form $(\mu_B, \mu_S, 1, \widehat{\sigma}(1))$. Recall that $\widehat{\sigma}(1) < s$ if and only if

$$\mu_S > \frac{(4-\gamma) - (3(3-\gamma) - (5-2\gamma)s\gamma)s\gamma}{(1-\gamma)s\gamma} \equiv \mu_S^*,$$

with $\mu_S^* > \mu_S^{M\sim U}$ and $\mu_S^* < 1$ for s sufficiently high, namely for $s > \frac{((5-2\gamma)-\sqrt{(1-\gamma)(5-2\gamma)})}{(5-2\gamma)\gamma} \equiv s_*$.

- When the sellers' cartel is flexible, a defection from the sellers' cartel has three possible consequences, depending on the magnitude of ε_S. (i) When $\beta_{M\sim U}^{\mu_S-\varepsilon_S} < s$, or equivalently when $\mu_S - \varepsilon_S > \mu_S^{M\sim U}$, and when the sellers' cartel continues to respond to $\beta = 1$ by setting $\widehat{\sigma}^{\mu_S-\varepsilon_S}(1) < s$, the deviation is profitable. (ii) When $\beta_{M\sim U}^{\mu_S-\varepsilon_S} < s$, or else when $\varepsilon_S < \mu_S - \mu_S^{M\sim U}$ but $\widehat{\sigma}^{\mu_S-\varepsilon_S}(1) \geq s$, being $\varepsilon_S \geq \mu_S - \mu_S^*$ then the sellers' cartel sets a quantity equal to s. Then, cartel members have individual payoff given by

$$\frac{\Pi_S(1, \widehat{\sigma}^{\mu_S}(1))}{s\mu_S} = \frac{(1-(1-\mu_S)s\gamma)(5-2\gamma) + (1-\gamma) - 2\sqrt{(1-\gamma)(5-2\gamma)(1-(1-\mu_S)\gamma s)}}{4s\mu_S}$$

before the defection, which is always greater than the outsiders' individual payoff, $\pi_S^S(1, s) = \gamma\frac{(4-\gamma)(1-\gamma s)-(1-\gamma)\gamma s}{4(1-\gamma s)}$. (iii) When $\beta_{M\sim U}^{\mu_S-\varepsilon_S} \geq s$, or equivalently when $\varepsilon_S \geq \mu_S - \mu_S^{M\sim U}$, the sellers' cartel sets $\sigma(1) = s$ and no deviation is profitable. Notice that, in order for the sellers' cartel to be flexible, it must be that $\mu_S \leq \overline{\mu}$, which is always the case when $\overline{\mu} \geq 1$ or else $\gamma \leq \overline{\gamma}$. When $\gamma > \overline{\gamma}$, instead, inequality $\mu_S^* < \overline{\mu}$ needs to hold, which happens if and only if

$$s > \frac{\left(89\gamma^2 - 59\gamma^3 + 12\gamma^4 - 52\gamma + 16\right) - (1-\gamma)\sqrt{\left(112\gamma^6 - 888\gamma^5 + 2713\gamma^4 - 4008\gamma^3 + 2992\gamma^2 - 1152\gamma + 256\right)}}{2\gamma^3(3-2\gamma)(5-2\gamma)} \equiv s_{**},$$

with $s_{**} > s_*$ if and only if $\gamma > \overline{\gamma}$. Therefore, the profile $(\mu_B, \mu_S, 1, \widehat{\sigma}(1))$ is ε-stable for any $\mu_B \leq \min\left\{\mu_B^{I\sim M}, \mu_B^{I\sim U}\right\}$, for any $s > \max\{s_*, s_{**}\}$, any $\mu_S^* < \mu_S \leq \overline{\mu}$ and for any $\varepsilon_S \geq \mu_S - \mu_S^*$.

[30] As in the case of symmetric trade, when $\sigma_{I\sim U}^{\mu_B-\varepsilon_B} = s$, the best response of the cartel is not unique, being $\beta^{\mu_B-\varepsilon_B}(s) = \{s, 1\}$. When $\sigma_{I\sim U}^{\mu_B-\varepsilon_B} = s$, a deviation might be profitable if the buyers' cartel switches from s to 1.

- When the sellers' cartel is radical, the profile $(\mu_B, \mu_S, 1, \widehat{\sigma}(1))$ represents an ε-stable market outcome if $\mu_S - \mu_S^{I\sim U} < \varepsilon_S < \mu_S - \max\{\overline{\mu}, \mu_S^*\}$ and $\mu_B \leq \min\{\mu_B^{I\sim M}, \mu_B^{I\sim U}\}$.

Acknowledgments This research has mostly been carried out while the authors were visiting the Institute for Policy Analysis at the University of Toronto, whose hospitality is gratefully acknowledged. We acknowledge financial support from the Catalan Government (grants 2000BEAI200120 to Burani and 2000SGR-0054 to Ponsati) and the Spanish Ministry of Education (grants PB98-970 and PR2000-0143 to Ponsati).

References

Bloch F, Ghosal S (2000) Buyers' and sellers' cartels on markets with indivisible goods. Rev Econ Des 5:129–147
Bogomolnaia A, Jackson MO (2002) The stability of hedonic coalition structures. Games Econ Behav 38:201–230
Burani N, Zwicker W (2003) Coalition formation games with separable preferences. Math Soc Sci 45:27–52
D'Aspremont C, Jacquemin A, Gabszewicz JJ, Weymark J (1983) The stability of collusive price leadership. Can J Econ 16:17–25
Donsimoni MP, Economides NS, Polemarchakis HM (1986) Stable cartels. Int Econ Rev 27:317–336
Galbraith JK (1952) American capitalism: the concept of countervailing power. Houghton Mifflin, Boston, MA
Horn H, Wolinsky A (1988) Worker substitutability and patterns of unionisation. Econ J 98:484–497
Inderst R, Wey C (2003) Bargaining, mergers, and technology choice in bilaterally oligopolistic industries. RAND J Econ 34:1–19
Lagos R (2000) An alternative approach to search frictions. J Political Econ 108:851–873
McAfee P, McMillan J (1992) Bidding rings. Am Econ Rev 82:579–599
Moreno D, Wooders J (2002) Prices, delay and the dynamics of trade. J Econ Theory 104:304–339
Ponsati C (2004) Search and bargaining in large markets with homogeneous traders. Contribut Theor Econ 4:1. http://www.bepress.com/bejte/contributions/vol4/iss1/art1
Ray D, Vohra R (1999) A theory of endogenous coalition structures. Games Econ Behav 26:286–336
Rubinstein A, Wolinsky A (1985) Equilibrium in a market with sequential bargaining. Econometrica 53:1133–1150
Scherer F, Ross D (1990) Industrial market structure and economic performance, 3rd edn. Houghton Mifflin, Boston, MA
Snyder CM (1996) A dynamic theory of countervailing power. RAND J Econ 27:747–769
Stigler GJ (1954) The economist plays with blocks. Am Econ Rev 44:7–14
Stole LA, Zwiebel J (1996) Organizational design and technology choice under intrafirm bargaining. Am Econ Rev 86:195–222
von Ungern-sternberg T (1996) Countervailing power revisited. Int J Indus Org 14:507–520
Wooders J (1997) Equilibrium in a Market with intermediation is walrasian. Rev Econ Des 3:75–89