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ORIGINAL PAPER

Arbitration systems and negotiations

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Abstract We consider a model of bargaining by concessions where agents can terminate negotiations by accepting the settlement of an arbitrator. The impact of pragmatic arbitrators—that enforce concessions that precede their appointment—is compared with that of arbitrators that act on principle—ignoring prior concessions. We show that while the impact of arbitration always depends on how costly that intervention is relative to direct negotiation, the range of scenarios for which it has an impact, and the precise effect of such impact, does change depending on the behavior—pragmatic or on principle—of the arbitrator. Moreover the requirement of mutual consent to appoint the arbitrator matters only when he is pragmatic. Efficiency and equilibrium are not aligned since agents sometimes reach negotiated agreements when an arbitrated settlement is more efficient and vice versa. What system of arbitration has the best performance depends on the arbitration and negotiation costs, and each can be optimal for plausible environments.

Keywords Arbitration · Bargaining · Concessions · Negotiations

JEL Classification C72 · C79

1 Introduction

Arbitration is an extended procedure of dispute resolution by which confronted parties submit to the decision of a third party. It is widely used in divorce proceedings, to settle

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grievances in union-management contracts, for the dissolution of partnerships, and in international trade. The use of arbitration when direct negotiation fails is often included as a clause in contracts and sometimes it is imposed by law.

To address the effect of arbitration over negotiations, bargaining games in the shadow of arbitration must be explored. While the literature on arbitration—both empirical and theoretical—is substantial,[1] most models in this literature do not contemplate arbitration and bargaining as alternatives that agents chose along a negotiation process. The notable exceptions are Compte and Jehiel (1995, 2004) and Manzini and Mariotti (2001).

Arbitration may be viewed as an outside option that bargainers have along the negotiation process. The literature on bargaining with outside options[2] explores the outcomes of non-cooperative bargaining games when agents can exit the negotiation and obtain an external payoff. In this literature, the decision to exit the negotiation can be taken unilaterally by the bargainers and it yields payoffs that are independent of the actions taken during the negotiation process. To think about arbitration, both assumptions may have drawbacks. It is thus important to clarify their distinct implications, and to study models in which they are relaxed.

The assumption that parties can unilaterally exit the negotiation and impose the use of arbitration is justified only in environments where, prior to beginning negotiations, parties commit to allow each other this possibility. Unilateral exit is thus a reasonable assumption to model bargaining under compulsory arbitration—a fairly common institutional arrangement for industrial relations in the USA, or in international trade disputes within the WTO where the complaining country may request the establishment of an arbitration panel if previous consultations fail. In other cases, however, parties negotiate without this prior commitment, and the decision to call an arbitrator requires mutual consent.[3] Manzini and Mariotti (2001) sharply make the point that the voluntary nature of arbitration is of great consequence. They propose a model of bargaining à la Rubinstein where the decision to call an arbitrator is a joint outside option[4] that both agents must agree to take, showing that outcomes are dramatically different from those of games where outside options are unilateral. The settlements imposed by their arbitrators, however, are independent of the negotiation history.

The assumption that arbitration outcomes are history independent must also be scrutinized. While some arbitrators do act on principle—imposing a fair settlement, independently of the concessions that precede their appointment—there is strong empirical evidence that this is not usually the case.[5] More often than not, arbitrators do pay attention to the events that precede their appointment, and act pragmatically imposing

[1] See Farber and Bazerman (1986), Gibbons (1988) and Kalai and Rosenthal (1979).

[2] See Shaked and Sutton (1984), Shaked (1994) and Ponsatí and Sakovics (1998).

[3] In Spain and the UK, for example, labour conflicts are resolved by an arbitrator only with the consent of both parties. See Manzini and Mariotti (2001) for a thorough discussion. In international commercial arbitration, the International Chamber of Commerce establishes that if the contract in dispute does not contain an ICC arbitration clause, parties can conclude a written ICC agreement to arbitrate. See Casella (1996) and Craig et al. (1990) for a description of the International Chamber of Commerce.

[4] See Manzini and Mariotti (2002) for a general disccussion not specifically focussed to arbitration.

[5] Ashenfelter and Bloom (1984), Ashenfelter (1987), Bloom (1986) and Farber and Bazerman (1986).

state dependent settlements that maintain previous concessions. Compte and Jehiel (2004) discuss concession bargaining games where players can unilaterally impose an arbitrated agreement at which payoffs depend on the concessions accumulated prior to exit. They show that endogenous outside options of this kind promote gradualism in the process of concessions, and thus delay and inefficiency.[6]

This paper explores concession bargaining games with different systems of arbitration in the background. We consider two agents that take turns at offering each other concessions, that cannot be claimed back. Negotiations are costly because each round of concessions takes time and players are impatient. When this game is played without an arbitrator in the background, the outcome is inefficient, agents agree at the Rubinstein shares with one period of delay.[7] When an arbitrator is present, the negotiation process can be terminated at any point by appointment of the arbitrator. Arbitration is costly as well, either directly, because a payment to the arbitrator consumes a portion of the surplus, or indirectly in terms of delay or other implementation frictions.[8]

There are two rules to appoint the arbitrator, and two norms of conduct by the arbitrator:

1. (a) Under unilateral arbitration, players do not need their opponent's approval to bring in the arbitrator; (b) under consensus arbitration they do.
2. (a) Pragmatic arbitrators, enforce concessions that take place prior to their appointment and split the contested surplus;[9] (b) arbitrators that act on principle impose a fair settlement regardless of prior concessions.

Each combination—unilateral pragmatic arbitration, pragmatic arbitration by consensus, unilateral arbitration on principle and arbitration on principle by consensus—constitutes an arbitration system. The alternating concession scheme, combined with an arbitration system, specifies a the game of complete information that is dominance solvable; the unique (subgame perfect) equilibrium can be obtained by (finite) iterative deletion of weakly dominated actions. For a wide range of the parameters the unique equilibrium is such that either the game terminates in arbitration immediately or a negotiated agreement occurs in two steps of concession. Focussing attention to these parameter configurations, we can provide a tractable explicit characterization of equilibrium outcomes, which allows a comparative assessment of the different

[6] The main message of Compte and Jehiel (2004) is general and important for many other applications, since their results apply to a very general class of games admitting several specifications and interpretations. Their previous unpublished work, Compte and Jehiel (1995), presents a discussion focussed to arbitration.

[7] This is in common with Admati and Perry (1991) and Compte and Jehiel (2004). See the latter for a convincing argument that this is an appropriate assumption to model negotiations with arbitration in the background.

[8] For example, the costs of arbitration of the International Chamber of Commerce, include the fees and expenses of the arbitrators and the ICC administrative costs. For a claim of $1,000,000 the maximum cost for three arbitrators is $113,925 and for a claim of 100,000,000 it is $610,125. In most of its international cases the duration of the arbitration procedure is at least one year. In the dispute settlement procedure of the WTO, parties do not pay any amount for the use of the pannel and the maximum time to settle a dispute is one year and three months.

[9] This is a reduced form for Conventional Arbitration, where arbitrators tend to compromise between the positions of the two parties. Equal shares of the contested surplus might also be the expected outcome of Final Offer Arbitration—where the arbitrator is constrained to choose one of the final two offers.

arbitration systems. We show that while the impact of arbitration always depends on its cost relative to the cost of direct negotiations, the range of scenarios for which it has an impact and the precise effect of such impact changes drastically depending on whether the arbitrator acts on principle or pragmatically. Moreover, for pragmatic arbitrators the requirement of consensus also matters.

When arbitrators are pragmatic, there are three possible equilibrium outcomes: If the cost of arbitration is high, agents ignore arbitration and reach agreement as they would in the game without arbitrator. At the other extreme, for cost of arbitration relatively low, then the arbitrator is appointed at the beginning of the game. In between there is a range of moderate costs where arbitration is not used, but it does have impact in the negotiated outcome; negotiated shares—no longer the Rubinstein shares—approach the potential arbitrated settlement. The scenarios at which each of these three outcomes prevail depend on whether arbitration is unilateral or by consensus. In contrast, when arbitrators act on principle they do not have an impact on negotiated agreements: either they are appointed immediately or they are irrelevant. Furthermore, the outcome that prevails is independent of the requirement of mutual consent.[10]

We show that efficiency and equilibrium outcomes are not aligned. There are scenarios where a negotiated outcome is the more efficient and players appoint the arbitrator, and scenarios where arbitration is efficient and yet, in equilibrium, players negotiate an agreement. Both types of inefficiency may arise when the arbitrator acts on principle or when is pragmatic. Whether one type of arbitrator or another is more effective in promoting efficiency depends on the arbitration and negotiation costs, so that none of the three arbitration systems can be dismissed as generally dominated by the others. Given a specific configuration, our analysis allows to assess precisely the cost or benefits of policies promoting changes in the appointment rules, or advocating changes the conduct of arbitrators. In real-life negotiations these inefficiencies are important. For example, The WTO accounts that, by July 2005, only 130 of the nearly 332 cases has reached the full panel process. Most of the rest have either been notified as "settled out of court" or remain in a prolonged consultation phase-some since 1995.

The remainder of the paper is organized as follows. Section 2 presents the model. The unique equilibrium when arbitrators are pragmatic is discussed in Sects. 3 and 4, that address, respectively, unilateral appointment and consensus. Section 5 considers arbitrators that act on principle. Section 6 compares the relative performance of the different systems. Conclusions are gathered in Sect. 7.

2 The model

Two players, $i = 1, 2$, bargain to share one unit of surplus. Negotiations take place over time and players are risk neutral and impatient. The different games that we consider combine general negotiation rules with a specification of the arbitration system. Negotiation rules, arbitration systems, outcomes and payoffs are described next.

[10] This result depends crucially on the concession nature of the bargaining game. As Manzini and Mariotti (2001) show, when agents are not constrained to maintain their concessions, an arbitrator appointed by consent that acts on principle may have great impact.

1. The negotiation rules: each period $t = 0, 1, 2, \ldots$ players may offer each other, in alternating order with player 1 moving first, mutual concessions; or they may appoint the arbitrator. Thus, at each t, and given the bargaining state (x_1, x_2, X), $0 \leq x_i \leq 1, 0 \leq x_1 + x_2 \leq 1, X = 1 - x_1 + x_2$ indicating the cumulative concession to each player in periods 0 to $t - 1$ and the contested surplus, player i must either offer to concede a non-negative additional portion of the surplus $C_i \in [0, X]$, or she can move to appoint the arbitrator.
2. Arbitration systems: an arbitration system specifies the appointment rule and the conduct of the arbitrator. There are two rules of appointment: in unilateral arbitration the game ends when a player moves to appoint the arbitrator; under consensus arbitration, after i moves to appoint the arbitrator, j must accept or reject. The arbitrator conduct specifies a partition of the surplus, (A_1, A_2), $A_i \geq 0$, $A_1 + A_2 = 1$. A pragmatic arbitrator appointed in state (x_1, x_2, X) respects concessions and splits the remaining surplus, thus $A_i = x_i + \frac{X}{2}$. When the conduct of the arbitrator is on principle $A_i = \frac{1}{2}$.
3. Outcomes and Payoffs: The game terminates when a player concedes all the contested surplus or the arbitrator is appointed. Otherwise after i moves first at t the game continues with j moving first at $t + 1$. Perpetual disagreement yields a zero payoff to both players. Under a negotiated agreement $(x, 1 - x, t)$ each player enjoys the accumulated concessions at the date of agreement, i.e. payoffs are $\left(\delta^t x, \delta^t (1 - x)\right)$, where $0 < \delta < 1$. Upon appointment of the arbitrator in period t at state (x_1, x_2, X), the arbitrator prescribes a split of the surplus into shares (A_1, A_2), and the cost of arbitration is incurred. Hence, payoffs upon an arbitrated termination are

$$\left(\delta^t \alpha A_1, \delta^t \alpha A_2\right),$$

where $0 < \alpha < 1$. A straight forward interpretation is that the arbitrator charges a direct fee proportional to the total surplus.[11] Even if the arbitrator does not charge a fee, arbitration will still be costly as long as it consumes resources or time.[12]

The extensive forms are displayed Figs. 1 and 2.

The present games are infinite horizon bargaining games of complete information. In spite of their close relationship to the standard bargaining games of alternating proposals, there are important differences that it is worthwhile to clarify. A first and fundamental difference is that players cannot claim back what they concede. Consequently, after each positive concession, the set of continuation strategies available to the players changes because the possible partitions of the surplus becomes smaller. Moreover, since strategy profiles where the first mover concedes the whole surplus

[11] Hence, individual costs of arbitration are proportional to the total share that each agent obtains. This is unlike in Compte and Jehiel (1995).

[12] In this later case, we treat the costs of arbitration as delay costs. That is, while $1 - \delta, \delta = \exp(-r)$, measures the cost imposed by a one period of delay in the negotiations, a share of the surplus obtained under an arbitrated outcome has a cost $1 - \alpha, \alpha = \exp(-rh)$, where h is the real time interval of delay imposed by the arbitrator. Note that $\alpha \leq \delta$ if and only if $h \geq 1$, i.e. if and only if arbitration takes longer than one round of bargaining.

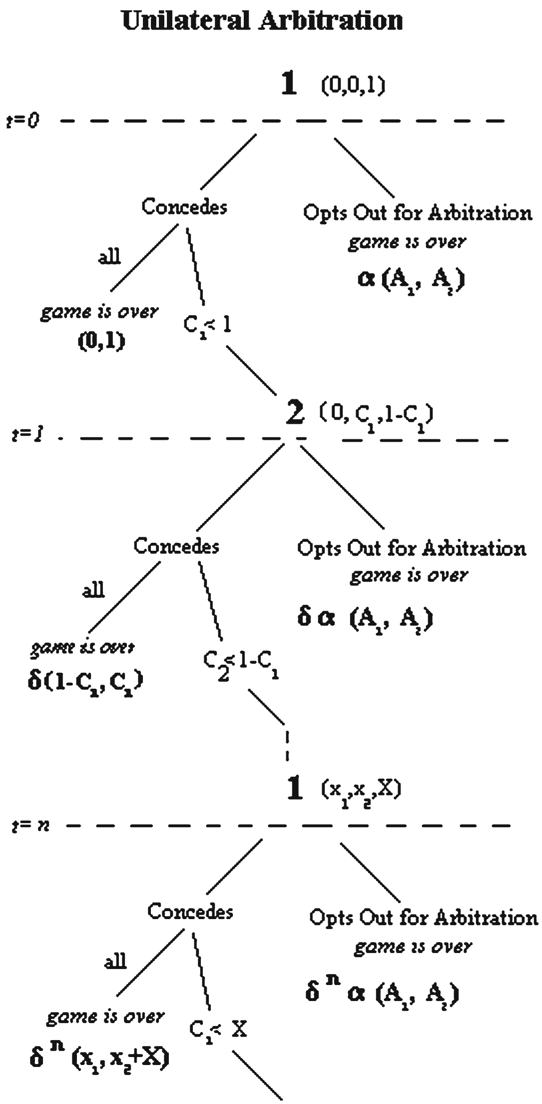

Fig. 1 Extensive form of the concession game with unilateral arbitration

can easily be ruled out as equilibria, negotiated agreements take at least one period. Finally, observe that proposing to appoint a pragmatic arbitrator is never equivalent to a concession $C_i = \frac{X}{2}$. Appointing the arbitrator terminates de game, and the costs of arbitration are incurred, while a partial concession $\frac{X}{2}$ leads to a continuation game at $t+1$ where the contested surplus is $1 - x_1 - x_2 - \frac{X}{2}$. If the appointment rule requires consensus, moreover, the opponent's rejection of arbitration prompts a continuation game at $t+1$ leaving the bargaining state unchanged.

Strategies specify actions at each subgame (a concession or the move to appoint the arbitrator; and, under consensus, thresholds to approve arbitration), and the set

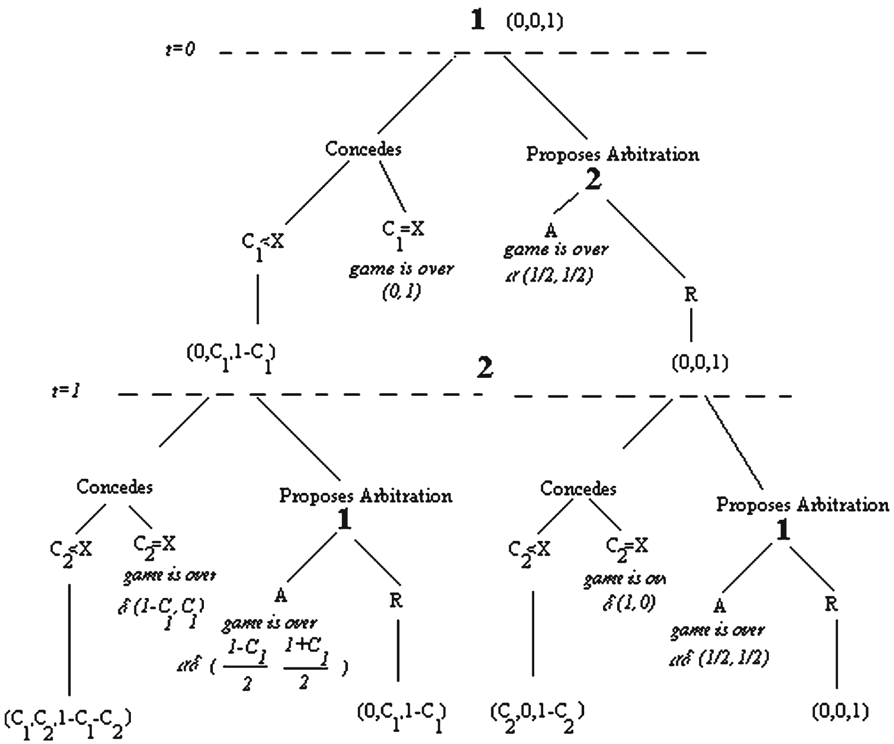

Fig. 2 Extensive form of the concession game with arbitration by consensus

of available actions is constrained by the state of the game. An equilibrium will be a profile of strategies that constitute a subgame perfect equilibrium.

In general, strategies may be extremely complex since actions at any subgame may depend arbitrarily on the entire history of actions up to that point, and the set of histories is large. However the bargaining state summarizes all information of a history that is payoff relevant to a player's choice, and for each state (x_1, x_2, X) a unique optimal action can be identified by sequential elimination of dominated actions.

Before we proceed to characterize equilibria in bargaining under arbitration it is useful to review the concession game in the absence of arbitration. Without arbitration, only one player moves at each round and she can either concede the rest of the pie, X, or make a partial concession $C_i \in [0, X]$. This game is discussed by Admati and Perry (1991), its unique equilibrium outcome yields agreement at the standard Rubinstein partition, attained with one period of delay.

Proposition 1 *No arbitration (Admati and Perry): in the absence of arbitration, in equilibrium, player* 1 *concedes* $\frac{\delta}{1+\delta}$ *and player* 2 *concedes the rest,* $\frac{1}{1+\delta}$*, in the following period.*

Proof See Admati and Perry (1991), Proposition 5.1. □

Without arbitration players concede up to the point where the opponent, given that she is impatient, is willing to terminate the game by conceding what is left. Since payoffs are only realized upon agreement, players do not benefit from the concessions they receive until the game ends. Therefore a player that has been granted a concession becomes effectively more impatient, delay is more costly for her that than for an opponent that has still nothing assured. If the first concession is large enough the optimal response is to terminate by conceding the rest of the pie. The minimal concession assures such response is the responder's share of the Rubinstein partition.

We now turn our attention to the effect that arbitration has in the preceding concession game. We start by analyzing a game under pragmatic arbitrators, that impose state dependent settlements, under the assumption that they can be appointed unilaterally.

3 Unilateral pragmatic arbitration

When agents can appoint the arbitrator unilaterally, arbitration is an outside option. Consequently, analyzing the equilibrium behavior of players under unilateral arbitration parallels the analysis of a bargaining game with outside options. The crucial insight is that outside options are not always relevant, and this is likewise with arbitration. Furthermore, when the outside option is of endogenous value varying at the different states of bargaining—as it will be in the present set up—its relevance is more delicate than that of fixed outside options.

Pragmatic arbitrators pay some attention to the history of negotiation that proceeds their appointment and prescribe settlements that are state dependent. Thus arbitration is indeed an option of endogenous value. Precisely, this endogenous value is

$$\alpha A_i\,(x_1, x_2, X) = \alpha \left(x_i + \frac{X}{2} \right);$$

that is, the accumulated concessions received prior to arbitration are enforced while the contested surplus is split equally, and the cost of arbitration is incurred.

At any subgame only one agent moves, and so she controls the rate at which payoffs are discounted. If she concedes, payoffs are discounted by δ; if the arbitrator is appointed, payoffs are discounted by α.

To describe equilibrium profiles we will simply specify the optimal action at each bargaining state. This characterization is given in Lemmas 2 to 5 that examine, in turn, all the state configurations that might arise along play of the game. The detailed proofs are relegated to the appendix. They involve tedious but otherwise straightforward sequential deletion of dominated actions, showing that for each state under consideration, the appropriate action survives uniquely.

The following notation simplifies the exposition. Given a state (x_1, x_2, X) we denote as C_i^N the concession of Player i that gives Player j an accumulated concession equivalent to the first mover payoff in the game without arbitration, that is,

$$C_i^N + x_j = \frac{\delta}{1+\delta}.$$

Similarly, we denote as C_i^A the concession of Player i that gives Player j an accumulated concession at which she is indifferent between terminating the game with total concession or with arbitration, that is,

$$C_i^A + x_j = \alpha \left(x_j + \frac{X + C_i^A}{2} \right).$$

We start by examining the optimal action at states where the active player has received accumulated concessions that exceed the present value of obtaining all remaining surplus in one period of delay. If the cost of arbitration is sufficiently high, she terminates the game by a total concession; otherwise the arbitrator is appointed.

Lemma 2 *In states where* $x_1 \geq \delta(x_1 + X)$ *the equilibrium action of Player* 1 *is as follows*:

1. *If* $\alpha \leq \frac{2\delta}{\delta+1}$ *Player* 1 *makes a concession* X.
2. *If* $\alpha > \frac{2\delta}{\delta+1}$ *Player* 1 *imposes arbitration in states where* $\alpha(x_1 + \frac{X}{2}) > x_1$; *otherwise she concedes* X.

Let us now examine the optimal actions when the active player faces an opponent that, upon continuation, will be in the situation of Lemma 2.

Lemma 3 *In states where* $x_1 < \delta(x_1 + X)$ *and* $x_2 \geq \delta(x_2 + X)$ *the equilibrium action of Player* 1 *is as follows*:

1. *If* $\alpha \leq \frac{2\delta}{\delta+1}$ *Player* 1 *does not to concede anything.*
2. *If* $\alpha > \frac{2\delta}{\delta+1}$ *Player* 1 *makes a concession* C_1^A *in states where* $\delta(x_1 + X - C_1^A) \geq \alpha(x_1 + \frac{X}{2})$; *otherwise she imposes arbitration.*

Our next result addresses optimal actions at states where no agent has reached the situation of Lemma 2, but one of the two players has received concessions that exceed $\frac{\delta}{1+\delta}$ (that is, the payoff that the first mover attains in the absence of arbitration). For the sake of tractability we limit attention parameters where it can be assured that an optimal action either terminates the game or leaves it at a state where the opponent will terminate it. This requires that costs of arbitration and negotiation are not simultaneously too low; $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$ is a sufficient condition.[13]

Lemma 4 *Assume that* $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$. *In states where* $x_i < \delta(x_i + X)$, $i = 1, 2$, *and either* $x_1 \geq \frac{\delta}{1+\delta}$, *or* $x_2 \geq \frac{\delta}{1+\delta}$, *in equilibrium Player* 1 *chooses an action that induces at most one period of delay. This action is as follows*:

1. *When* $x_1 \geq \frac{\delta}{1+\delta}$: (i) *If* $\alpha \leq \frac{2\delta}{2\delta+1}$ *a concession* X. (ii) *If* $\frac{2\delta}{2\delta+1} < \alpha \leq \frac{2\delta}{1+\delta}$ *a concession* X *if* $x_1 \geq \alpha(x_1 + \frac{X}{2})$; *otherwise to impose arbitration.* (iii) *If* $\frac{2\delta}{\delta+1} < \alpha$ *a concession* C_1^A *if* $\delta(x_1 + X - C_1^A) \geq \alpha(x_1 + \frac{X}{2})$; *otherwise to impose arbitration.*

13 A complete characterization addressing scenarios where the equilibrium is a k-step process of gradual concessions a la Compte and Jehiel (2004) is a tedious exercise that adds no further insight into the issues that concern us.

2. *When* $x_2 \geq \frac{\delta}{1+\delta}$: (i) *No concession if* $\alpha \leq \frac{2\delta}{1+2\delta}$. (ii) *If* $\frac{2\delta}{2\delta+1} < \alpha \leq \frac{2\delta}{1+\delta}$ *a concession* C_1^A *if* $\delta(x_1 + X - C_1^A) \geq \alpha(x_1 + \frac{X}{2})$ *and else to impose arbitration.* (iii) *If* $\frac{2\delta}{\delta+1} < \alpha$ *a concession* C_1^A *if* $\delta(x_1 + X - C_1^A) \geq \alpha(x_1 + \frac{X}{2})$; *otherwise to impose arbitration.*

Considering states where no agent has yet reached accumulated concessions beyond the first mover's payoff in the absence of arbitration completes the exploration of optimal actions. We maintain the constraint to parameters such that $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$.

Lemma 5 *Assume that* $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$. *In states such that* $x_i < \frac{\delta}{1+\delta}$, $i = 1, 2$, *in equilibrium Player* 1 *chooses an action that induces at most one period of delay. This action is as follows*:

1. *If* $\alpha \leq \frac{2\delta}{2\delta+1}$ *Player* 1 *makes a concession* C_1^N.
2. *If* $\frac{2\delta}{2\delta+1} < \alpha$ *Player* 1 *makes a concession* $C_1 = \text{Max}\left[C_1^N, C_1^A\right]$ *whenever* $\delta(x_1 + X - C_1) \geq \alpha(x_1 + \frac{X}{2})$; *otherwise she imposes arbitration.*

With a full characterization of the optimal action at each possible bargaining state the full characterization of the equilibrium outcomes is straightforward. It suffices to observe that the optimal actions at the initial state (0, 0, 1) are given in Lemma 5. They must necessarily yield either an arbitrated termination or a negotiated agreement that occurs in two steps of concession.

For each state that satisfies the conditions of Lemma 5, the different optimal action scenarios generate a partition in a subset of parameters. For the initial state (0, 0, 1), it is immediate to check that the induced partition is as follows. The parameters (α, δ) for which $C_i^N = \frac{\delta}{1+\delta}$ is the optimal action must lie in the set

$$H = \left\{(\alpha, \delta) \text{ such that } \alpha \leq \frac{2\delta}{1+2\delta}\right\};$$

and the optimal action is $C_i^A = \frac{\alpha}{2-\alpha}$ at set of parameters

$$M = \left\{(\alpha, \delta) \text{ such that } \frac{2\delta}{1+2\delta} < \alpha \leq \min\left\{\frac{2}{2+\delta}, 1 + 2\delta - \sqrt{1+4\delta^2}\right\}\right\}.$$

The arbitrator is appointed if costs are low, i.e. $(\alpha, \delta) \in L$ where L is defined as

$$L = \left\{(\alpha, \delta) \text{ such that } 1 + 2\delta - \sqrt{1+4\delta^2} < \alpha < 2(1-\delta^2)\right\}.$$

Figure 3 displays these sets.

The main result of this section, characterizing the unique equilibrium outcome for some parameters α and δ, is now immediate from Lemmas 5 and 4.

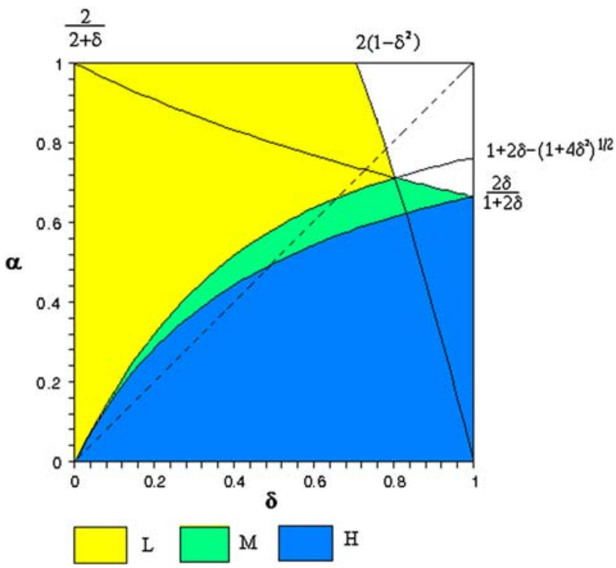

Fig. 3 *M* and *H* are the parameter regions where in equilibrium a negotiated partition is implemented and *L* is the area where arbitration prevails in equilibrium under the unilateral arbitration regime

Proposition 6 Unilateral pragmatic arbitration. *Let* $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$. *When a pragmatic arbitrator—allocating shares* $A_i\,(x_1, x_2, X) = x_i + \frac{X}{2}$*—can be appointed unilaterally, in the unique equilibrium there is at most one period of delay. Arbitration prevails if it has low cost,* $(\alpha, \delta) \in L$*, and this outcome occurs at* $t = 0$*. Otherwise a negotiated agreement is reached at* $t = 1$*. The negotiated partition* $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ *prevails under high cost arbitration,* $(\alpha, \delta) \in H$*; when* $(\alpha, \delta) \in M$ *the split is* $\left(\frac{2(1-\alpha)}{2-\alpha}, \frac{\alpha}{2-\alpha}\right)$.

It is now immediate to observe that the potential efficiency gain offered by arbitration is not always realized; and that it is also possible that arbitration decreases efficiency.

Corollary 7 Equilibrium and efficiency are not aligned. *Efficiency demands that arbitration prevails if and only if* $\alpha \geq \delta$. *When* $\delta \leq \frac{2}{3}$*, and* $\delta \leq \alpha \leq 1+2\delta-\sqrt{1+4\delta^2}$ *the potential gains from arbitration are not realized. When* $\delta > \frac{2}{3}$ *and* $1+2\delta-\sqrt{1+4\delta^2} \leq \alpha \leq \min\left\{\delta, 2(1-\delta^2)\right\}$ *arbitration prevails and it imposes a loss of efficiency.*

Figure 4 displays the parameter regions for which the potential gains of arbitration are unrealized, UG^u, and where arbitration induces efficiency losses, EL^u.

We now turn to examine the consequences of requiring consensus to appoint the arbitrator.

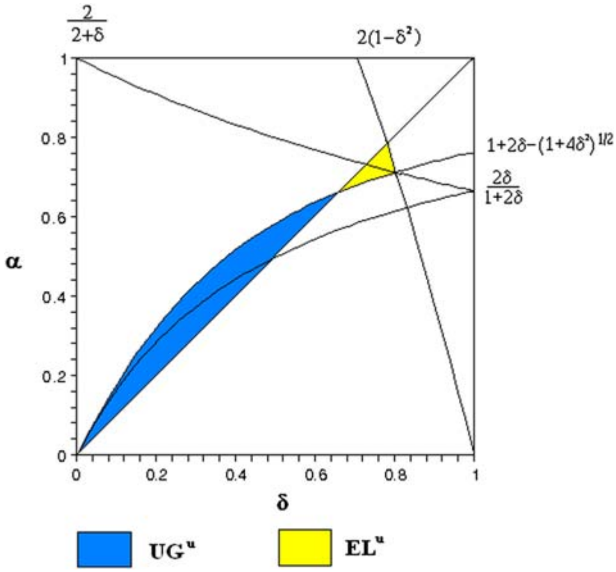

Fig. 4 UG^u is the parameter region where efficiency demands arbitration and players negotiate and EL^u is the region for which players use unilateral arbitration whenever negotiation is a more efficient procedure to solve the dispute

4 Pragmatic arbitration by consensus

When consensus is required to terminate the negotiation and bring in the arbitrator, arbitration becomes a joint outside option. Consequently, when the first mover wishes to appoint the arbitrator, her bargaining power—that again arises from her control of the rate at which payoffs are discounted—is limited by the veto power of the opponent. In equilibrium, arbitration is rejected if the responder expects greater payoffs from continuing the negotiation than from the arbitrated settlement.

One may think that, if a player has the right to veto arbitration, she will use this right when the arbitrated outcome is unfavorable, neutralizing the presence of the arbitrator. Contrary to this intuition, we show that the presence of an arbitrator in the background remains a strong influence on the bargaining outcome, even when the consent of both parties is required.

The characterization of equilibria in the present environment is given through Lemmas 8 to 11 that establish optimal actions in a way analogous to Lemmas 2 to 5 of the previous section. Since proposing arbitration is a dominated action in states where the optimal reply of the opponent is to reject it, we can safely omit acceptance rules in describing the optimal actions of players at each state of the game. Thus we will simply specify the optimal action of the first mover at each bargaining state. The detailed proofs are in the appendix. They follow along the same arguments used to prove the analogous results in the previous section. They are, however, a bit more involved since the constraints imposed by the rule of consensus must be taken into consideration. The action leading to arbitration is taken in fewer states than in the game with unilateral arbitration: not only the costs of arbitration must be sufficiently low, in addition, the approval of Player 2 must be granted.

Our first observation, however, is that things do not change in states where the responding agent has received accumulated concessions that exceed the present value of obtaining all remaining surplus in one period of delay. In these states, if the proposal of arbitration is not a dominated action, its acceptance is assured. Hence the actions that the proposer takes in equilibrium are not affected by the requirement of consensus. Whatever action was optimal under unilateral arbitration remains optimal when arbitration needs consensus.

Lemma 8 *Consider states where $x_2 \geq \delta(x_2 + X)$. If opting out to arbitration is the equilibrium action of Player* 1 *under unilateral arbitration, the equilibrium action under consensus arbitration is to propose arbitration. Otherwise, the equilibrium action of Player* 1 *under consensus or unilateral arbitration coincide.*

Let us now examine the optimal actions when Player 1 is in the situation of Lemma 8.

Lemma 9 *In states where $x_1 \geq \delta(x_1 + X)$ and $x_2 < \delta(x_2 + X)$ the equilibrium action of Player* 1 *is as follows*:

1. *If $\alpha \leq \frac{2\delta}{1+\delta}$ Player* 1 *concedes X.*
2. *If $\alpha > \frac{2\delta}{1+\delta}$ Player* 1 *proposes arbitration whenever $\alpha(x_1 + \frac{X}{2}) > x_1$ and $x_2 \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$, and otherwise she concedes X.*

We now consider states where one of the two players has received concessions that exceed $\frac{\delta}{1+\delta}$. As in the previous section, we limit attention to the parameter region where it can be assured that equilibrium actions terminate the game in at most two steps. A sufficient condition is that $\alpha \notin [\frac{2}{2+\delta}, 2\delta^2]$.

Lemma 10 *Assume $\alpha \notin [\frac{2}{2+\delta}, 2\delta^2]$. In states where $x_i < \delta(x_i + X), i = 1, 2$ and either $x_1 \geq \frac{\delta}{1+\delta}$ or $x_2 \geq \frac{\delta}{1+\delta}$ the equilibrium action of Player 1 induces at most one period of delay.*

1. *If $x_1 \geq \frac{\delta}{1+\delta}$, the the equilibrium action of Player* 1 *is as follows*:
   (a) *A concession X if either (i) $\alpha \leq \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$, or (ii) $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha \leq \frac{2\delta}{1+\delta}$ with $x_1 \geq \alpha(x_1 + \frac{X}{2})$, or (iii) $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha \leq \frac{2\delta}{1+\delta}$ with $x_1 < \alpha\left(x_1 + \frac{X}{2}\right)$ and $x_2 < \frac{1-\alpha}{1-\delta^2}\left(\frac{2\delta^2}{\alpha} - 1\right)$.*
   (b) *To propose arbitration if either (i) $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha \leq \frac{2\delta}{1+\delta}$ with $x_1 < \alpha(x_1 + \frac{X}{2})$ and $x_2 \geq \frac{1-\alpha}{1-\delta^2}\left(\frac{2\delta^2}{\alpha} - 1\right)$ or (ii) max$\left\{\frac{2\delta}{1+\delta}, 2\delta^2\right\} < \alpha$ and $\alpha\left(x_1 + \frac{X}{2}\right) > \delta(x_1 + X - C_1^A)$.*
   (c) *A concession C_1^A if max$\left\{\frac{2\delta}{1+\delta}, 2\delta^2\right\} < \alpha$ and $\alpha\left(x_1 + \frac{X}{2}\right) \leq \delta(x_1 + X - C_1^A)$.*
2. *If $x_2 \geq \frac{\delta}{1+\delta}$, the the equilibrium action of Player* 1 *is as follows*:

(a) *Concede nothing if either* (i) $\alpha \leq \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$ *or* (ii) $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha \leq \frac{2\delta}{1+\delta}$ *with* $x_2 \geq \alpha(x_2 + \frac{X}{2})$, *or (iii)* $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha < \frac{2\delta}{1+\delta}$ *with* $x_2 < \alpha\left(x_2 + \frac{X}{2}\right)$ *and* $x_1 < \frac{1-\alpha}{1-\delta^2}\left(\frac{2\delta^2}{\alpha} - 1\right)$.

(b) *Propose arbitration if* $\max\left\{\frac{2\delta}{1+\delta}, 2\delta^2\right\} < \alpha$ *and* $\alpha\left(x_1 + \frac{X}{2}\right) > \delta(x_1 + X - C_1^A)$.

(c) *A concession* C_1^A *if (i)* $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha \leq \frac{2\delta}{1+\delta}$ $x_2 < \alpha\left(x_2 + \frac{X}{2}\right)$ *and* $x_1 \geq \frac{1-\alpha}{1-\delta^2}\left(\frac{2\delta^2}{\alpha} - 1\right)$ *or (ii)* $\max\left\{\frac{2\delta}{1+\delta}, 2\delta^2\right\} < \alpha$ *and* $\alpha\left(x_1 + \frac{X}{2}\right) \leq \delta(x_1 + X - C_1^A)$.

And the characterization of optimal actions at parameter configurations where $\alpha \notin [\frac{2}{2+\delta}, 2\delta^2]$ is completed with our next result.

Lemma 11 *Assume* $\alpha \notin [\frac{2}{2+\delta}, 2\delta^2]$. *In states such that* $x_i < \frac{\delta}{1+\delta}$ $i = 1, 2$ *the equilibrium action of Player* 1 *induces at most one period of delay. This action is as follows.*

1. *Assume* $\alpha \leq \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$, *then the equilibrium action of Player* 1 *is a concession* C_1^N.
2. *Assume* $\alpha > \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$.
   (a) *Let* $x_i \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1), i = 1, 2$. *The equilibrium action of Player* 1 *is a concession* $C_1 = \text{Max}\left\{C_1^A, C_1^N\right\}$ *whenever* $\alpha(x_1 + \frac{X}{2}) \leq \delta(x_1 + X - C_1)$, *and otherwise she proposes arbitration.*
   (b) *Let* $x_i < \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ *for some* i. *The equilibrium action of Player* 1 *is to propose arbitration whenever* $\alpha(x_1 + \frac{X}{2}) > \delta(x_1 + X - C_1)$ *and* $\alpha(x_2 + \frac{X}{2}) > \frac{\delta^2}{1+\delta}$, *and otherwise she concedes* $C_1 = \text{Max}\left\{C_1^A, C_1^N\right\}$.

To characterize the unique equilibrium outcome it suffices to consider the partition of the set of parameters induced by the optimal action scenarios of Lemma 11 at the initial state (0, 0, 1). The parameters (α, δ) for which $C_i^N = \frac{\delta}{1+\delta}$ is the optimal action are in the set

$$H^C = H \cup \left\{(\alpha, \delta) \text{ such that } \frac{2\delta}{1+2\delta} < \alpha \leq \min\left\{2\delta^2, \frac{2}{2+\delta}\right\}\right\}.$$

And the optimal action is $C_i^A = \frac{\alpha}{2-\alpha}$ at parameters that lie in

$$M^C = M / \left\{(\alpha, \delta) \text{ such that } \alpha < 2\delta^2\right\}.$$

When $(\alpha, \delta) \in L^C$, the optimal action is to propose arbitration, and

$$L^C = L / \left\{(\alpha, \delta) \text{ such that } \alpha < 2\delta^2\right\}.$$

These sets are displayed in Fig. 5.

The unique equilibrium outcome is now immediate from Lemmas 11 and 10.

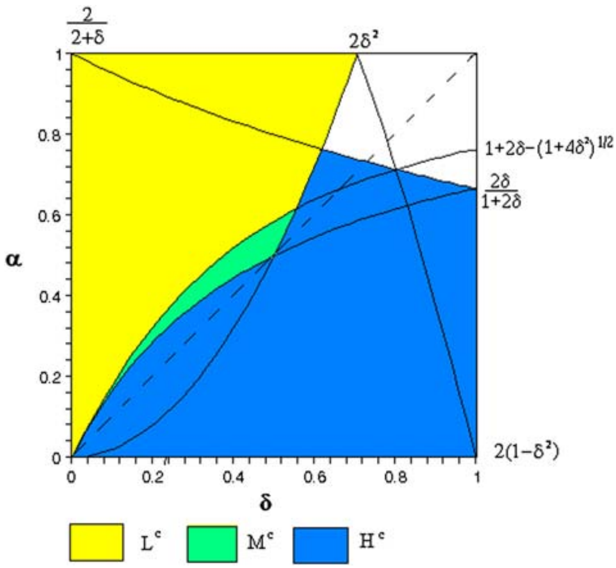

Fig. 5 M^C and H^C are the set of parameters where in equilibrium a negotiated partition is implemented and L^C is the set of parameters for which arbitration prevails in equilibrium under the arbitration by consensus regime

Proposition 12 Pragmatic arbitration by consensus. *Let* $\alpha \notin [\frac{2}{2+\delta}, 2\delta^2]$. *When a pragmatic arbitrator—allocating shares* $A_i = x_i + \frac{X}{2}$*—is appointed by consensus, in the unique equilibrium there is at most one period of delay. Arbitration prevails if it has low cost,* $(\alpha, \delta) \in L^C$*, and this outcome occurs at* $t = 0$*. Otherwise a negotiated agreement is reached at* $t = 1$*. The negotiated partition* $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ *prevails under high cost arbitration,* $(\alpha, \delta) \in H^C$*; otherwise, when* $(\alpha, \delta) \in M^C$ *the split is* $\left(\frac{2(1-\alpha)}{2-\alpha}, \frac{\alpha}{2-\alpha}\right)$.

The following is now immediate.

Corollary 13 Too many negotiated agreements. *Arbitration is not used inefficiently since a negotiated agreement surely prevails when* $\alpha < \delta$*. Still, efficiency and equilibrium remain unaligned, since* $H^C \cup M^C$ *contains a substantial region where arbitration is the superior procedure to solve the dispute,* $\alpha > \delta$ *, and yet players reach a negotiated agreement foregoing the potential gains* $\alpha - \delta$.

Figure 6 displays the parameter region where the potential gains of arbitration remain unrealized, UG^c. Pragmatic arbitrator appointed by consensus cannot induce an efficiency loss, $EL^c = \emptyset$.

5 Arbitration on principle

To complete our exploration we now consider arbitrators that act on principle, ignoring the negotiation process that precedes their appointment, and imposing a fair settlement

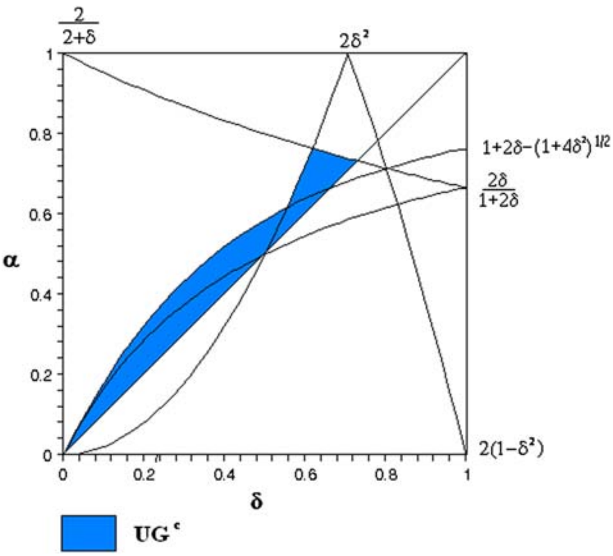

Fig. 6 The coloured area UG^c corresponds to the set of parameters where inefficient negotiation prevails

independently of the state that the negotiations have reached. That is, at all states (x_1, x_2, X), the arbitrated share of both players is

$$A_i = \frac{1}{2}.$$

When bargainers interact with such an arbitrator in the background, either they ignore her, or they waste no time in appointing her. It turns out that there is a simple threshold that separates scenarios for which agents opt out to arbitration, below that threshold, arbitration is irrelevant and agents behave as in Proposition 1. Furthermore, the consensus required to bring in the arbitrator is irrelevant, the same outcome prevails if agents can do it unilaterally or when they need the opponent's approval. This results are stated formally as Proposition 14 that follows. The proof is in the "Appendix".

Proposition 14 Arbitration on principle. *Assume that the arbitrated settlement is fixed at* $\left(\frac{1}{2}, \frac{1}{2}\right)$. *The same equilibrium outcome prevails independently of whether the decision to opt out to arbitration is unilateral or by consensus. If* $\alpha \leq \frac{2\delta}{1+\delta}$ *then a negotiated agreement* $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ *is reached at* $t = 1$*; otherwise arbitration prevails at* $t = 0$.

Fig. 7 displays the range of inefficiencies induced by arbitrators that act on principle. The set UG^p contains parameter configurations where the equilibrium leaves unrealized gains.

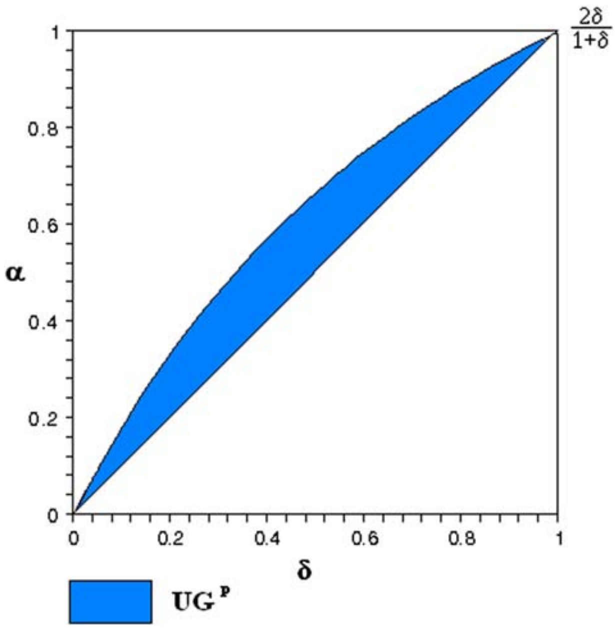

Fig. 7 UG^p is the parameter region where the potential gains of the arbitration on principle procedure remain unrealized because in equilibrium players negotiate

6 Comparing arbitration systems

With the characterization of equilibrium outcomes that prevail under each arbitration system we may now compare their relative performance, and evaluate the effect or changes in the system. The performance of each arbitration system depends on the likelihood of the different cost configurations.

Consider arbitration systems with a unilateral appointment rule and consider the effects of changes in the conduct of the arbitrator. The set of parameters where the change towards a pragmatic arbitrator affects efficiency is PP (see Fig. 8). The range of scenarios in which arbitration is efficiently used is greatly increased by the pragmatic conduct of the arbitrator. When $\alpha > \delta$ and $\alpha \in \left[1 + 2\delta - \sqrt{1 + 4\delta^2}, \min\left\{\frac{2\delta}{1+\delta}, 2(1 - \delta^2)\right\}\right]$, an arbitrator that acts on principle is not appointed while if he acts pragmatically he will be called.

Observe that when the arbitrator is pragmatic and mutual consent is required the range of scenarios at which agents reach negotiated agreements is greater than when is appointed unilaterally. Now assume that arbitrators act pragmatically consider the effects of policy changes in the appointment rule. When the change is from unilateral to consensus, the result is that arbitration ceases to be used. Conversely, as the consensus requirement is dropped, arbitration becomes more widely used. For parameters in UC, requiring consensus is clearly damaging to welfare, since the change eliminates the efficiency gain offered by arbitration; conversely allowing unilateral appointment increase welfare. In contrast, for parameters in CU, requiring consensus

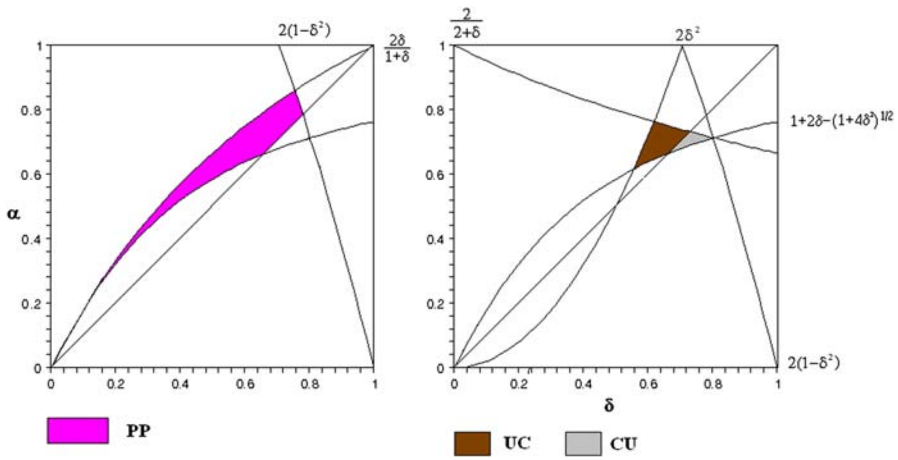

Fig. 8 Change in regimes and gains in efficiency

increases welfare, since this eliminates the inefficient use of arbitration that prevails under unilateral appointment. But of course, in this scenario, the same positive effect is attained if arbitration is no longer available. Thus, when arbitrators are pragmatic, if the change from unilateral appointment to mutual consent is regarded as a useful policy, then it must be that arbitrators are inefficient.

7 Conclusions

Aiming to explore the impact of different arbitration systems on negotiations, we have characterized the equilibrium of concession bargaining games with arbitration in the background. We summarize our findings in the following three points:

1. Arbitration might alter the negotiated partition of the surplus relative to the situation in which it is unavailable. This occurs only if the arbitrated partition of the surplus is endogenous and the relative cost of arbitration is not too high, since arbitration turns irrelevant when it is excessively costly. When arbitration is relevant, the negotiation positions of the players approach those sustained by the arbitrator and the first mover advantage is reduced. If the cost of arbitration is sufficiently low players immediately resort to arbitration, and an equal split of the surplus prevails.
2. The requisite of mutual consent is of great consequence if the arbitrator is pragmatic, but it is irrelevant when the arbitrator acts on principle.
3. Arbitration cannot assure full efficiency. What system of arbitration promotes the greater gain in efficiency depends on the expected negotiation and arbitration costs. When there are potential efficiency gains from arbitration, a pragmatic arbitrator induces better outcomes than one acting on principle. For a pragmatic arbitrator, the requirement of mutual consent increases efficiency only if arbitration does not offer potential efficiency gains.

Appendix

Unilateral arbitration

Proof of Lemma 2 Player 1 chooses one of the following alternatives: (a) concede X and receive a payoff of x_1; (b) concede nothing obtaining at most $\delta(x_1 + X)$ in the continuation; (c) concede $0 < C_1 < X$, obtaining at most $\delta(x_1 + X - C_1)$; (d) Opt out to arbitration, which pays $\alpha\left(x_1 + \frac{X}{2}\right)$.

Since $x_1 \geq \delta(x_1 + X) > \delta(x_1 + X - C_1)$ the optimal action is to concede X provided that $x_1 > \alpha(x_1 + \frac{X}{2})$. For $x_1 \geq \delta(x_1 + X)$ the inequality $x_1 \geq \alpha(x_1 + \frac{X}{2})$ is equivalent to $\alpha \leq \frac{2\delta}{\delta+1}$. And if $\alpha > \frac{2\delta}{\delta+1}$ then player 1 opts out in states where $\alpha(x_1 + \frac{X}{2}) > x_1$ since opting out dominates making any concession. □

Proof of Lemma 3 In scenario (1) Player 2 faces the situation analyzed in Lemma 2 (1); hence her optimal action is to concede X. Now consider Player 1: if she concedes nothing, player 2 will concede X in the following period, and 1 obtains $\delta(x_1 + X)$. This payoff is greater than what 1 would get if she concedes X since $\delta(x_1 + X) > x_1$. Likewise $\delta(x_1 + X) > \delta(x_1 + X - C_1)$ implying that any $C_1, 0 < C_1 < X$ is dominated as well. To rule out arbitration simply observe that to call the arbitrator is in Player 1's interest only if $\alpha(x_1 + \frac{X}{2}) \geq \delta(x_1 + X)$, and note that $x_1 < \delta(x_1 + X)$ and $\alpha \leq \frac{2\delta}{1+\delta}$ imply that the preceding inequality is impossible. To check 2) note that by Lemma 2 (2) Unless she receives a sufficient concession Player 2 will opt out in the continuation. Thus the action of Player 1 must be either to opt out to arbitration right away or to concede something that prevents arbitration, the minimal concession achieves that is C_1^A. Thus Player 1 opts out it if and only if $\alpha(x_1 + \frac{X}{2}) \geq \delta(x_1 + X - C_1^A)$ and concedes C_1^A otherwise. Notice that $C_1^A = 0$ for the states satisfying $x_2 > \alpha(x_2 + \frac{X}{2})$. □

Proof of Lemma 4 The equilibrium action of Player 1 must be one of the following alternatives:

1. (a) concede X and obtain a payoff x_1;
   (b) impose arbitration and obtain a payoff $\alpha\left(x_1 + \frac{X}{2}\right)$;
   (c) concede $C_1 \geq \widetilde{C}_1$ and obtain $\delta(x_1 + X - C_1)$, where $\widetilde{C}_1$ is the minimal concession that leads the game to a state where Lemma 3 - 1 applies and 2 concedes all the contested surplus at $t + 1$ (i.e. $x_2 + \widetilde{C}_1 = \delta(x_2 + X)$;
   (d) concede $0 \leq C_1 < \widetilde{C}_1$.

Claim 1 For states where $x_1 \geq \frac{\delta}{1+\delta}$, (c) is dominated by (a) for all (α, δ). Note that $x_1 \geq \frac{\delta}{1+\delta}$ is equivalent to $x_1 \geq \delta(x_1 + X - \widetilde{C}_1)$ and $\delta(x_1 + X - \widetilde{C}_1) > \delta(x_1 + X - C_1)$. □

Claim 2 For states where $x_1 \geq \frac{\delta}{1+\delta}$, if $\alpha \leq \max\left\{2(1-\delta^2), \frac{2}{2+\delta}\right\}$ *then either* (a) *or* (b) *dominate* (d).

Among states such that $x_i < \delta(x_i + X)$ $i = 1, 2$ and $x_1 \geq \frac{\delta}{1+\delta}$ consider those that satisfy $x_1 \geq \alpha(x_1 + \frac{X}{2})$. Since in these states (a) dominates (b) we need to prove that (a) dominates (d) as well. Consider $C_1 < \widetilde{C}_1$ that leads to a state where Lemma 2.1 applies. Player 2 concedes nothing at her turn and Player 1 obtains $\delta^2 x_1$; that is dominated by (a). Hence C_1 must be small enough so that the subsequent bargaining state still lies in the set of bargaining states that we are presently examining. Following such $C_1 < \widetilde{C}_1$, 1 can expect from 2 at most $\widetilde{C}_2$, the concession that leaves Player 1 ready to finish the game at her next turn. Thus $C_1 < \widetilde{C}_1$ pays 1 at most $\delta^2(x_1 + \widetilde{C}_2)$. Substituting $\widetilde{C}_2 = \delta(x_1 + X) - x_1$, we obtain that $x_1 \geq \delta^2(x_1 + \widetilde{C}_2)$ is equivalent to $x_1 \geq \delta^3(x_1 + X)$ and therefore (a) dominates (c) in states that satisfy the later inequality. A recursive argument completes the proof of the claim. Given δ there is a natural number such that $\delta^{n+1}(x_1 + X) < x_1 < \delta^n(x_1 + X)$. Assume that in bargaining states satisfying $x_1 < \delta(x_1 + X)$, $x_2 < \delta(x_2 + X)$, $x_1 \geq \frac{\delta}{1+\delta}$ and $\delta^{n+1}(x_1 + X) < x_1$, conceding X dominates any other partial concession. In states such that $x_1 < \delta(x_1 + X)$, $x_2 < \delta(x_2 + X)$, $x_1 \geq \frac{\delta}{1+\delta}$ and $x_1 < \delta^n(x_1 + X)$ the concession that player 1 can expect in the following from player 2 is no greater than C_2 such that $x_1 + C_2 = \delta^n(x_1 + X)$. Hence, the expected payoff of from a concession smaller than X, is no greater than $\delta^2(x_1 + C_1) = \delta^{2+n}(x_1 + X) < \delta^{n+1}(x_1 + X) < x_1$ so that 1 is better off conceding X.

Consider now states such that $x_1 < \alpha(x_1 + \frac{X}{2})$ where clearly (b) dominates (a). We will prove that (b) dominates (d). In these states we can define the minimal concession C_2^A such that leads to one of the states considered in the previous paragraph, that is, C_2^A that satisfies $x_1 + C_2^A = \alpha(x_1 + C_2^A + \frac{X - C_2^A}{2})$. If player 1 makes a concession $C_1 < \widetilde{C}_1$, then, at best she can expect a concession from player 2 of C_2^A. Then we need to prove that $\alpha(x_1 + \frac{X}{2}) > \delta^2(x_1 + C_2^A)$. Notice that substituting C_2^A in $\delta^2(x_1 + C_2^A)$ we get $\frac{\alpha\delta^2}{2-\alpha}(x_1 + X)$. We will distinguish two cases:

Case 1 (α, δ) such that $\alpha \leq \frac{2}{2+\delta} = \mathrm{Max}\left\{\frac{2}{2+\delta}, 2(1-\delta^2)\right\}$. It is easy to prove that for such set of parameters the following inequality is satisfied $\frac{\delta}{1+\delta} > \frac{\alpha\delta^2}{2-\alpha}$. Then $\alpha(x_1 + \frac{X}{2}) > x_1 \geq \frac{\delta}{1+\delta} > \frac{\alpha\delta^2}{2-\alpha} \geq \frac{\alpha\delta^2}{2-\alpha}(x_1 + X)$.

Case 2 (α, δ) such that $\alpha \leq 2(1-\delta^2) = \mathrm{Max}\left\{\frac{2}{2+\delta}, 2(1-\delta^2)\right\}$. Then, $\alpha(x_1 + \frac{X}{2}) > \frac{\alpha}{2}(x_1 + \frac{X}{2}) \geq \frac{\alpha\delta^2}{2-\alpha}(x_1 + X)$.

This completes the proof of the claim. □

To establish the equilibrium actions for states where $x_1 \geq \frac{\delta}{1+\delta}$, it is now sufficient to compare the payoffs of (a) and (b) in each parameter region:

(i) $\alpha \leq \frac{2\delta}{1+2\delta}$: (a) dominates (b) because $x_1 \geq \alpha(x_1 + \frac{X}{2})$ for all $x_1 \geq \frac{\delta}{1+\delta}$. First note that $\alpha \leq \frac{2\delta}{1+2\delta}$, i.e. $\frac{1}{1+\delta} \leq \frac{2(1-\alpha)}{2-\alpha}$, implies that if $x_2 < \delta(x_2 + X)$, $x_1 < \delta(x_1 + X)$ and $x_1 \geq \frac{\delta}{1+\delta}$ then $x_1 \geq \alpha(x_1 + \frac{X}{2})$ since $(1-\alpha)x_1 \geq (1-\alpha)\frac{\delta}{1+\delta} \geq \frac{\alpha}{2(1+\delta)} \geq \frac{\alpha}{2}X$.

(ii) $\frac{2\delta}{2\delta+1} < \alpha$: In states where $x_1 \geq \alpha(x_1 + \frac{X}{2})$ (a) clearly dominates (b); on the other hand in states where $x_1 < \alpha(x_1 + \frac{X}{2})$ (b) is the preferred action of Player 1.

(iii) $\frac{2\delta}{\delta+1} < \alpha$: (b) dominates (a) since for all bargaining states such that $x_1 \geq \frac{\delta}{1+\delta}$ it is satisfied that $\alpha\left(x_1 + \frac{X}{2}\right) > x_1$.

This completes the proof of 1.

Let us now consider states where $x_2 \geq \frac{\delta}{1+\delta}$. The equilibrium actions follow easily taking into account that the equilibrium moves of Player 2 in the continuation are as described in point 1.

(i) $\alpha < \frac{2\delta}{1+2\delta}$: Player 1 will choose to concede nothing since, in the continuation, Player 2 concedes X and $x_1 < \delta(x_1 + X)$, $\delta(x_1 + X - C_1) < \delta(x_1 + X)$ and $\alpha(x_1 + \frac{X}{2}) < \delta(x_1 + X)$.

(ii) $\frac{2\delta}{1+2\delta} < \alpha < \frac{2\delta}{1+\delta}$: At these bargaining states, in the continuation, Player 2 either concedes X or calls the arbitrator. If at (x_1, x_2, X) Player 2 calls the arbitrator, then the choice of Player 1 will be between conceding C_1^A or calling the arbitrator. If Player 2 concedes X then, Player 1 will choose to concede nothing (that is, $C_1^A = 0$) since $\delta(x_1 + X) > \delta(x_1 + X - C_1)$ and $\delta(x_1 + X) > \alpha\left(x_1 + \frac{X}{2}\right)$.

(iii) $\frac{2\delta}{1+\delta} < \alpha$: Player 1 either concedes C_1^A or calls the arbitrator since conceding nothing is dominated, $\alpha(x_1 + \frac{X}{2}) > \frac{\alpha}{2}(x_1 + X) > \frac{\alpha\delta^2}{2-\alpha}(x_1 + X) = \delta^2(x_1 + C_2^A)$.

This completes the proof of 2. □

Proof of Lemma 5 We first show 1. If $\alpha \leq \frac{2\delta}{1+2\delta}$ a concession C_1^N assures to 1 a payoff $\frac{\delta}{1+\delta}$ (since Lemma 4.1 applies to Player 2 in the continuation so that she responds conceding the rest of the pie). To concede more than C_1^N is clearly dominated. To concede less is also dominated; in that case, at the new bargaining state, Player 1 cannot expect to receive more than C_2^N, such that $x_1 + C_2^N = \frac{\delta}{1+\delta}$ so that by conceding less than C_1^N, player 1 can get, at most, $\frac{\delta^3}{1+\delta} < \frac{\delta}{1+\delta}$. Arbitration is dominated by C_1^N provided that $\frac{\delta}{1+\delta} \geq \alpha\left(x_1 + \frac{X}{2}\right)$, which holds when $\alpha \leq \frac{2\delta}{2\delta+1}$; simply note that $\alpha\left(x_1 + \frac{X}{2}\right) \leq \alpha\left(\frac{\delta}{1+\delta} + \frac{1}{2}\left(1 - \frac{\delta}{1+\delta}\right)\right) = \frac{\alpha}{2}\frac{2\delta+1}{1+\delta}$ and that $\frac{\alpha}{2}\frac{2\delta+1}{1+\delta} \leq \frac{\delta}{1+\delta}$ for all $\alpha \leq \frac{2\delta}{2\delta+1}$.

Let us now establish 2. Consider first states such that $x_i < \frac{\delta}{1+\delta}$ $i = 1, 2$ and $\delta(x_1 + X - C_1) \geq \alpha\left(x_1 + \frac{X}{2}\right)$ where $C_1 = \text{Max}\left\{C_1^N, C_1^A\right\}$. Opting out is clearly dominated, since either $\alpha\left(x_1 + \frac{X}{2}\right) \leq \frac{\delta}{1+\delta} \leq \delta(x_1 + X - C_1^A)$ or $\alpha\left(x_1 + \frac{X}{2}\right) \leq \delta(x_1 + X - C_1^A) \leq \frac{\delta}{1+\delta}$. Moreover, while conceding $C_1^{'} > C_1$ is obviously dominated, a concession $C_1^{'} < C_1$ leads the continuation game to a state where player 2 either opts out to arbitration or concedes. If a concession $C_1^{'} < C_1$ leads the game to a state where player 2 opts out to arbitration, player 1 gets $\delta\alpha\left(x_1 + \frac{X - C_1^{'}}{2}\right)$, and $\delta\alpha\left(x_1 + \frac{X - C_1^{'}}{2}\right) < \alpha\left(x_1 + \frac{X}{2}\right) \leq \delta(x_1 + X - C_1)$. If $C_1^{'}$ leads the game a the state where player 2 concedes, player 1 gets, at most, $\max\left\{\frac{\delta^3}{1+\delta}, \delta^2(x_1 + C_2^A)\right\}$. We will now prove that this payoff is smaller than what is obtained by conceding C_1, provided that (α, δ) are such that $\alpha \leq \text{Max}\left\{\frac{2}{2+\delta}, 2(1-\delta^2)\right\}$. We distinguish two cases:

(a) $\max\left\{\frac{\delta^3}{1+\delta}, \delta^2(x_1 + C_2^A)\right\} = \delta^2(x_1 + C_2^A)$.

If $\frac{2}{2+\delta} < 2(1-\delta^2)$ we can establish that $\delta^2(x_1 + C_2^A) \leq \frac{\alpha\delta^2}{2-\alpha} < \frac{\alpha}{2}(x_1 + X) < \alpha(x_1 + \frac{X}{2}) < \delta(x_1 + X - C_1)$. And $2(1-\delta^2) \leq \frac{2}{2+\delta}$ implies the following: either $\delta^2(x_1 + C_2^A) \leq \frac{\alpha\delta^2}{2-\alpha} < \frac{2\delta(1-\alpha)}{2-\alpha} \leq \delta(x_1 + X - C_1^A)$, or $\delta^2(x_1 + C_2^A) = \frac{\alpha\delta^2}{2-\alpha}(x_1 + X) \leq \frac{\alpha\delta^2}{2-\alpha} < \frac{\delta}{1+\delta}$.

(b) $\max\left\{\frac{\delta^3}{1+\delta}, \delta^2(x_1 + C_2^A)\right\} = \frac{\delta^3}{1+\delta}$

Then either $\frac{\delta^3}{1+\delta} < \frac{\delta}{1+\delta}$, or $\frac{\delta^3}{1+\delta} < \frac{\alpha\delta^2}{2-\alpha} < \frac{2\delta(1-\alpha)}{2-\alpha} \leq \delta(x_1 + X - C_1^A)$.

Consider finally states such that $x_i < \frac{\delta}{1+\delta}$ $i = 1, 2$ and $\alpha(x_1 + \frac{X}{2}) \geq \delta(x_1 + X - C_1)$ where $C_1 = \text{Max}\left\{C_1^N, C_1^A\right\}$. We only need to prove that conceding less than C_1 is dominated. Again, if a concession $C_1^{'} < C_1$ leads the continuation game to a state where Player 2 opts out to arbitration, then Player 1 gets $\delta\alpha\left(x_1 + \frac{X - C_1^{'}}{2}\right) < \alpha(x_1 + \frac{X}{2})$, and if Player 2 concedes all in the continuation, Player 1 obtains, at most, $\max\left\{\frac{\delta^3}{1+\delta}, \delta^2(x_1 + C_2^A)\right\}$. Since arbitration dominates conceding C_1, then a) and b) can be used to prove that it also dominates conceding $C_1^{'}$. □

Arbitration by consensus

Proof of Lemma 8 It suffices to observe that the proposal of arbitration is always accepted by 2 whenever opting out is the optimal action of Player 1 under unilateral arbitration.

If $\alpha \leq \delta$ opting out is never optimal (see Lemmas 2.1, 3.1). And if $\alpha > \delta$ and Player 1 proposes arbitration, Player 2 will always accept since $\alpha(x_2 + \frac{X}{2}) > \delta x_2$. □

Proof of Lemma 9 If $\alpha \leq \frac{2\delta}{1+\delta}$ Player 1 concedes X since $x_1 \geq \alpha(x_1 + \frac{X}{2})$. And if $\alpha > \frac{2\delta}{1+\delta}$ there will be some bargaining states such that $x_1 < \alpha(x_1 + \frac{X}{2})$. Player 1 will propose arbitration and Player 2 will surely accept in the states such that $\alpha(x_2 + \frac{X}{2}) \geq \delta^2(x_2 + X)$ or $\alpha(x_2 + \frac{X}{2}) < \delta^2(x_2 + X)$ and $x_2 \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ since in both cases $\alpha(x_2 + \frac{X}{2}) \geq \delta^2(x_2 + X - C_2^A)$. Otherwise Player 2 rejects arbitration because at least she can get $\delta^2(x_2 + X - C_2^A)$ and then it is better to concede X. □

Proof of Lemma 10 For $\alpha \leq \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$ the same arguments used in the proof of Lemma 4 establish that the optimal action are either (a) to concede X and obtain a payoff of x_1, or (b) to propose arbitration, that (provided that 2 accepts it) yields payoff of $\alpha\left(x_1 + \frac{X}{2}\right)$.

When $\alpha \leq \frac{2\delta}{1+2\delta}$ (a) dominates (b) because $x_1 \geq \alpha(x_1 + \frac{X}{2})$. First note that $\alpha \leq \frac{2\delta}{1+2\delta}$, i.e. $\frac{1}{1+\delta} \leq \frac{2(1-\alpha)}{2-\alpha}$, implies that if $x_2 < \delta(x_2 + X)$, $x_1 < \delta(x_1 + X)$ and

$x_1 \geq \frac{\delta}{1+\delta}$ then $x_1 \geq \alpha(x_1 + \frac{X}{2})$ since $(1-\alpha)x_1 \geq (1-\alpha)\frac{\delta}{1+\delta} \geq \frac{\alpha}{2(1+\delta)} \geq \frac{\alpha}{2}X$. If $\frac{2\delta}{1+2\delta} < \alpha \leq \delta$ we find states for which $x_1 < \alpha(x_1 + \frac{X}{2})$. However, Player 2 rejects the arbitration proposal since $\alpha(x_2 + \frac{X}{2}) \leq \delta^2(x_2 + X)$.

If $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha < \frac{2\delta}{1+\delta}$, then arbitration is no longer unanimously dominated at all states under consideration. Total concession obviously dominates arbitration for states where $x_1 \geq \alpha(x_1 + \frac{X}{2})$. In states such that $x_1 < \alpha(x_1 + \frac{X}{2})$, Player 1 prefers arbitration over conceding X. Player 2 acceptance is guaranteed for states satisfying $x_2 \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ and arbitration is preferred to concede nothing since $\alpha(x_1 + \frac{X}{2}) \geq x_1 \geq \frac{\delta}{1+\delta} \geq \frac{\alpha\delta^2}{2-\alpha} \geq \delta^2(x_1 + C_2^A)$. Otherwise Player 1 concedes X, since the arbitration proposal will be rejected and another concession $C_1 \geq 0$ is a dominated alternative.

The result for scenario $\alpha > \max\left\{\frac{2\delta}{1+\delta}, 2\delta^2\right\}$ follows from Lemma 8. Simply observe that whenever Player 1 proposes arbitration, Player 2 accepts since $\alpha(x_2 + \frac{X}{2}) \geq \frac{\alpha}{2}(x_2 + X) > \delta^2(x_2 + X)$. This completes the proof of 1.

2. Follows from immediately from 1. □

Proof of Lemma 11 We begin with the proof of 1. A concession C_1^N assures to 1 a payoff $\frac{\delta}{1+\delta}$ provided that Lemma 10 1.—(i) applies to Player 2 in the continuation so that she responds conceding the rest of the pie. To rule out concessions $C_1 \neq C_1^N$ we argue as in the proof of Lemma 5. It remains to be checked that arbitration is dominated as well. Arbitration is dominated by C_1^N provided that $\frac{\delta}{1+\delta} \geq \alpha(x_1 + \frac{X}{2}) = \frac{\alpha}{2}(1 + x_1 - x_2)$ and that it suffices that $\frac{\delta}{1+\delta} \geq \frac{\alpha}{2}(1 + x_1)$ which is equivalent to $x_1 \leq \frac{\alpha - 2\delta(1-\alpha)}{\alpha(1+\delta)}$. Assume that $\alpha \leq \max\left\{\frac{2\delta}{1+2\delta}, \delta\right\}$ holds and $\alpha\left(x_1 + \frac{X}{2}\right) \geq \frac{\delta}{1+\delta}$, acceptance by Player 2 requires that $\alpha\left(x_2 + \frac{X}{2}\right) \geq \frac{\delta^2}{1+\delta}$. Both inequalities combined imply that $\alpha > \delta$. Hence, $\delta < \alpha < \frac{2\delta}{2\delta+1}$ but the later inequality contradicts that $\alpha\left(x_1 + \frac{X}{2}\right) \geq \frac{\delta}{1+\delta}$: Simply note that $\alpha\left(x_1 + \frac{X}{2}\right) \leq \alpha\left(\frac{\delta}{1+\delta} + \frac{1}{2}\left(1 - \frac{\delta}{1+\delta}\right)\right) = \frac{\alpha}{2}\frac{2\delta+1}{1+\delta}$ and that $\frac{\alpha}{2}\frac{2\delta+1}{1+\delta} < \frac{\delta}{1+\delta}$ for all $\alpha < \frac{2\delta}{2\delta+1}$.

Next we prove 2 (a). If $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha$ and $x_i \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ $i = 1, 2$. acceptance of the arbitration proposal is guaranteed since $x_2 \geq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$. By the same arguments used in the proof of Lemma 5 we prove that Player 1 concedes $C_1 = \text{Max}\left\{C_1^A, C_1^N\right\}$ whenever $\alpha(x_1 + \frac{X}{2}) \leq \delta(x_1 + X - C_1)$ or else she proposes arbitration.

Finally we prove 2(b). If $\max\left\{\frac{2\delta}{1+2\delta}, \delta\right\} < \alpha$ and $x_i < \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ for some i, propose arbitration whenever $\alpha(x_1 + \frac{X}{2}) > \delta(x_1 + X - C_1)$ and $\alpha(x_2 + \frac{X}{2}) > \frac{\delta^2}{1+\delta}$. Otherwise a concession $C_1 = \text{Max}\left\{C_1^A, C_1^N\right\}$. If $x_1 \leq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha} - 1)$ then $C_1^N = \text{Max}\left\{C_1^A, C_1^N\right\}$. Assume first that $\alpha(x_1 + \frac{X}{2}) \leq \frac{\delta}{1+\delta}$. A concession of C_1^N gives Player 1 a payoff of $\frac{\delta}{1+\delta}$ since Player 2 concedes X in the following period (Lemma 101) (iii). To concede more than C_1^N is clearly a dominated option, and to concede $C_1^{'} < C_1^N$ leads the game to a state where Player 2 concedes, at most, $C_2 = \text{Max}\left\{C_2^N, C_2^A\right\}$ or propose arbitration. In the case of concession, $\frac{\delta}{1+\delta} > \text{Max}\left\{\frac{\delta^3}{1+\delta}, \frac{\delta^2\alpha}{2-\alpha}\right\} \geq \delta^2(x_1+X-C_1^{'})$. And if Player 2 proposes arbitration, then $\alpha\delta(x_1+\frac{X-C_1}{2}) < \alpha(x_1+\frac{X}{2}) \leq \frac{\delta}{1+\delta}$. Assume now that $\alpha(x_1+\frac{X}{2}) > \frac{\delta}{1+\delta}$ and $\alpha(x_2+\frac{X}{2}) > \frac{\delta^2}{1+\delta}$. Then Player 1 will propose arbitration if Player 2 accepts it. If he rejects he will get at most $\max\left\{\frac{\delta^2}{1+\delta}, \delta^2(x_1+C_2^A)\right\}$ and $\alpha(x_2+\frac{X}{2}) > \delta^2(x_2+C_1^A)$ if $\alpha < 2(1-\delta^2)$ and $\alpha(x_2+\frac{X}{2}) > \frac{\delta^2}{1+\delta}$. Otherwise, Player 2 will reject the arbitration proposal and Player 1 will concede $C_1 = \text{Max}\left\{C_1^A, C_1^N\right\}$. If $x_2 \leq \frac{1-\alpha}{1-\delta^2}(\frac{2\delta^2}{\alpha}-1)$, the same argument applies for $C_1^A = \text{Max}\left\{C_1^A, C_1^N\right\}$. □

Arbitration on principle

Proof of Proposition 14 It is easy to check that when the payoffs from arbitration are fixed at $\frac{\alpha}{2}$, the optimal actions at each of the four possible state configurations are as stated in Claims 1 to 4.

Claim 1 Consider states where $x_1 \geq \delta(x_1+X)$. (A) Assume unilateral appointment. If $x_1 \geq \frac{\alpha}{2}$, then Player 1 concedes X; otherwise Player 1 opts out to arbitration. (B) Assume appointment by mutual consent. If $x_1 \geq \frac{\alpha}{2}$ or $\delta x_2 \geq \frac{\alpha}{2}$, then Player 1 concedes X; otherwise Player 1 proposes arbitration.

Claim 2 Consider states where $x_1 < \delta(x_1+X)$ and $x_2 \geq \delta(x_2+X)$. (A) Assume unilateral appointment. (A1) If $\frac{\delta}{1+\delta} \geq \frac{\alpha}{2}$ and $\delta(x_1+X) \geq \frac{\alpha}{2}$ Player 1 concedes nothing. (A2) If $\frac{\delta}{1+\delta} \geq \frac{\alpha}{2}$ and $\delta(x_1+X) < \frac{\alpha}{2}$ Player 1 opts out. (A3) And if $\frac{\delta}{1+\delta} \geq \frac{\alpha}{2}$ she opts out to arbitration. (B) Assume appointment by mutual consent. If $\delta x_2 \geq \frac{\alpha}{2}$ Player 1 concedes nothing; otherwise she proposes arbitration whenever $\delta(x_1+X) < \frac{\alpha}{2}$ or else she concedes nothing.

Claim 3 Consider states where $x_i < \delta(x_i+X), i = 1, 2$. (A) Assume unilateral appointment. (A1) Assume $x_1 \geq \frac{\delta}{1+\delta}$. (i) If $\frac{\delta}{1+\delta} \geq \frac{\alpha}{2}$ then Player 1 concedes X. (ii) If $\frac{\delta}{1+\delta} < \frac{\alpha}{2}$ then Player 1 concedes X whenever $x_1 \geq \frac{\alpha}{2}$, otherwise she opts out. (A2) Assume $x_2 \geq \frac{\delta}{1+\delta}$. (i) If $\alpha \leq \frac{2\delta}{1+\delta}$ then Player 1 concedes nothing whenever $\delta(x_1+X) \geq \frac{\alpha}{2}$ or else opts out to arbitration. (ii) If $\alpha > \frac{2\delta}{1+\delta}$ Player 1 opts out to arbitration. (B) Assume appointment by mutual consent. (B1) Assume $x_1 \geq \frac{\delta}{1+\delta}$. If $\alpha \leq \frac{2\delta}{1+\delta}$ then Player 1 concedes X; otherwise she proposes arbitration. (B2) Assume $x_2 \geq \frac{\delta}{1+\delta}$. Then Player 1 concedes nothing if $\alpha \leq \frac{2\delta}{1+\delta}$, otherwise she proposes arbitration.

Claim 4 Consider states such that $x_i < \frac{\delta}{1+\delta} i = 1, 2$. (A) Assume unilateral appointment. If $\alpha \leq \frac{2\delta}{1+\delta}$, then Player 1 concedes C_1^N; otherwise she opts out to arbitration. (B) Assume appointment by mutual consent. If $\alpha \leq \frac{2\delta}{1+\delta}$, then Player 1 concedes C_1^N; otherwise she proposes arbitration.

Proposition 14 is now immediate form Claims 3 and 4. □

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