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All in good time

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Abstract Why is issue-by-issue bargaining a common phenomenon, even though it disallows the beneficial trade-offs across issues that are possible when negotiating a global solution? We show that under asymmetric information, issue-by-issue bargaining has two attractive features. First, it avoids bundling a good deal on one issue with a bad deal on another issue, when the lack of gains to agreement on the latter is not a priori common knowledge. Second, it avoids the imposition of the asymmetric information inefficiency of "harder" issues on issues which turn to be "easier" to solve. That's why when the agenda is not imposed on the parties but is rather left for them to determine when negotiating, they may very well opt for issue-by-issue bargaining. We demonstrate this in a natural game where bargainers communicate, whenever they choose to do so, their willingness to discuss or make offers either on one of the issues or on a bundle of issues.

Keywords Issue-by-issue bargaining · Endogenous agenda · Asymmetric information

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1 Introduction

Negotiations addressing several issues often proceed by solving first easier issues, while harder issues are discussed in parallel but their resolution is postponed until their details get finalized. For example, the 1979 Egyptian–Israeli peace treaty drafted an autonomy plan for the Palestinians, but postponed the definition of its details and implementation to a later stage. In Ireland, the 1998 "Good Friday" agreement granted self-ruling to Northern Ireland in exchange for an appeal to decommission the paramilitary groups, but the details of this decommission continued to be discussed and implemented in stages, along many years. In the World Trade Organization (WTO), agreed-upon issues are implemented, while harder issues on the table are deferred to a later round of negotiations.

In the classical formulation of Nash a bargaining problem consists of selecting, from a given set of feasible outcomes, an agreement unanimously preferred to disagreement. In this set-up, it is clear that global bargaining offers (weakly) better prospects than separating the different issues that compose a bargaining problem: When the bargainers have different marginal rates of substitution across the issues, in global bargaining each player can give up something in one issue to get more in the other. Such trade-offs are precluded when the issues are discussed separately, and the bargaining set shrinks.

Nevertheless, practical advice from negotiation experts is mixed (see for example Raiffa 1982). Recognizing that global bargaining increases the feasible set of agreements, they recommend combining deals on different issues as a major instrument to generate win-win solutions. But they also offer heuristic arguments in favor of separation: that delay to resolve a difficult issue should not interfere with the execution of an agreement for an issue that is easier to resolve; or that separating issues allows parties to rely on different specialized agents for each issue, thus simplifying problems that are too complex.

These informal arguments in favor of separation are linked to information incompleteness, a pervasive phenomenon in actual negotiations. In this paper we introduce asymmetric information in a model of multi-issue bargaining and show that two new effects appear on stage, which may offset the picture in favor of separation. We thus substantiate, by a formal treatment, the above-mentioned *mixture* of practical advice, and highlight the causes underlying the merits of separation under incomplete-information. We do not claim that with asymmetric information separation is preferable under any bargaining protocol. Rather, we demonstrate the reasons why separation *may* lead to better outcomes, and why it may be endogenously chosen by the parties, as we often see in practice.

We consider two parties that must split two different pies, the issues. In order to demonstrate the effects favoring separation, we start by isolating them from the known effect that favors global bargaining. To this end, we restrict attention to a setting where the marginal rate of substitution across issues is the same for both parties. Under complete information, global bargaining would be just as effective as separate bargaining. With asymmetric information, when each party has a privately known cost to bear in order to consume her share of each pie, we show how separation may yield superior results, due to two distinct reasons.

The first reason is that there might be losses to agreement in one of the issues, without this being common knowledge at the outset. If the issues are negotiated separately, indeed no deal will be struck for this issue. However, if there are large gains to agreement in other issues and all issues are negotiated together, the final agreement will include the better-not-to-agree-upon issue in the package. The overall expected gains from the global agreement might therefore be reduced by the inclusion of this black-sheep issue. We demonstrate this in Sect. 3, where we assume that the two pies are of equal size, and that the (privately known) costs of each issue to each player are drawn independently from a uniform distribution. Addressing each issue separately through the optimal mechanism of Myerson and Satterthwaite (1983)[1] delivers expected gains above the first-best benefits of global bargaining, let alone the expected gains of any mechanism for global bargaining that satisfies the incentive constrains.

The second reason favoring separation is that the inefficiencies inherent to equilibrium might "add up" when separate bargaining is replaced by global bargaining. This latter effect may be operative independently of the former. In Sect. 4 we propose a natural description of the bargaining process where this phenomenon is very powerful indeed. We show that when negotiations proceed as in the game introduced by Wang (2000),[2] the equilibrium outcomes of separate bargaining are preferred by both players to those of global bargaining, *for every realization of the private information.*

Thus, when designing the agenda for bargaining under incomplete information, separating the issues can benefit both parties, who may therefore unanimously agree on such a separation in the first place. This may thus be a consideration in institutional design of mediation, diplomacy and markets.

Oftentimes, however, the agenda is neither exogenously set for the bargainers by some institution, nor do they discuss it explicitly at the outset. Rather, the agenda then emerges endogenously with the negotiators' behavior. Will this endogenous agenda give rise to separate or global bargaining? In Sect. 5 we propose an extension of the Wang (2000) game, where players choose to

[1] That is, the direct revelation mechanism that maximizes ex-ante gains subject to (interim) incentive-compatiblility and individually rationality.

[2] Wang's game is a simplification of the bargaining game of Cramton (1992). Both Cramton (1992) and Wang (2000) extend the Admati and Perry (1987) one-sided incomplete information game with finitely many types to two-sided asymmetric information with a continuum of types. All these games generalize the Rubinstein (1982) alternating-offers protocol by allowing the bargainers to move at their will, imposing only some minimal elapse before any counter-move. We concentrate on the equilibrium behavior in the limit when this elapse vanishes.

make offers either on one of the issues or on both. We show that there exists an equilibrium at which issues are negotiated separately on the equilibrium path, with the threat of global bargaining off the equilibrium path, while there does not exist an equilibrium with the reverse structure.

Our results resonate with the literature which claims that, in complete-information alternating-offers bargaining, separating issues may be preferable in the presence of "frictions", modeled as a positive risk of breakdown if and when an offer is rejected. John and Raith (2001) show this under the assumption that the probability of breakdown for each separate issue is smaller than that for a combination of issues. In the approach we offer here, it is a result rather than an assumption that the friction or inefficiency due to asymmetric information over each single issue is smaller than in global bargaining. Chen (2006) proposes a game where the order and packaging of issues arises endogenously and shows that, with a positive risk of breakdown, negotiating issues separately may yield larger (expected) gains: In global bargaining, the players face the risk of not being the first to offer, while in separate bargaining they can trade-off the first-mover advantage across the issues. In the game we analyze in Sect. 5, in contrast, we focus on the limit case in which the first-mover advantage disappears, so the order of moves is not the driving force for our result. Moreover, in our analysis this order is endogenous to the equilibrium rather than pre-determined.

Agendas, their impact on bargaining outcomes and their endogenous determination, are the main concern in the formal literature on bargaining over multiple issues. Kalai (1977), Ponsati and Watson (1997) and O'Neill et al. (2004) present a variety of formulations for multiple-issue bargaining problems and discuss the agenda manipulability of solutions with an axiomatic approach. The other strand of the literature (Fershtman 1990; Busch and Horstmann 1997a,b; Inderst 2000; In and Serrano 2003; Chen 2006) is concerned with the extension of the Rubinstein (1982) approach to multiple-issue environments. Bac and Raff (1996) and Busch and Horstmann (1999) discuss games with private information in which the choice of the agenda is a signaling device.

Our message should not be interpreted as a normative advice. In fact, even in the absence of trade-offs across issues, if an active benevolent mediator can regulate the bargaining process, the global approach might be best. Jackson and Sonnenschein (2006)[3] supply precise normative arguments favoring collective decisions over packages: They show that the efficiency loss imposed by informational constraints decreases when a bargaining problem is resolved jointly with many copies of itself, provided that agents are severely limited on the kind of offers they can make on bundles, disallowing extreme offers on too many issues. Nevertheless, bargaining often takes place under rather unstructured procedures, and extreme offers on all the issues may occur. For these situations, we provide a positive theory that explains why separating issues is so common.

[3] See also Cohn (2004).

2 Bargaining problems

The bargaining problem is formalized as follows. Two parties $i = 1, 2$ negotiate how to split two pies of a known size $A_k > 0, k = 1, 2$. For each k each party i has a privately known cost c_{ik} she has to bear in order to consume her share of the pie k. The vector of costs $(c_{11}, c_{12}, c_{21}, c_{22})$ is drawn from some distribution with a support $\left[\underline{c}_{11}, \bar{c}_{11}\right] \times \left[\underline{c}_{12}, \bar{c}_{12}\right] \times \left[\underline{c}_{21}, \bar{c}_{21}\right] \times \left[\underline{c}_{22}, \bar{c}_{22}\right]$. This support is assumed to be common knowledge, but otherwise the parties may entertain incongruent beliefs about the details of the distribution. We assume that $\underline{c}_{1k} + \underline{c}_{2k} < A_k$, for $k = 1, 2$ so that there are conceivable net gains to sharing each of the pies – at most

$$G_k = A_k - \underline{c}_{1k} - \underline{c}_{2k}.$$

In *separate bargaining* the two pies are bargained concurrently and independently. An outcome of separate bargaining specifies a date (possibly infinite) of agreement and shares for each pie. Both parties discount future gains at the instantaneous rate r. Upon an outcome where agent i receives a share $x_{ik,}$ $0 \leq x_{ik} \leq A_k, x_{1k} + x_{2k,} = A_k$, at dates t_k, $0 \leq t_k \leq \infty$, $k = 1, 2$ her payoff is

$$e^{-rt_1} (x_{i1} - c_{i1}) + e^{-rt_2} (x_{i2} - c_{i2}).$$

In *global bargaining* the two issues are bargained together; agreements apply, simultaneously, to the bundle of both issues. Thus, the bundle of the two pies is bargained together as a single pie of size $A = A_1 + A_2$. In this case an outcome specifies shares of the global pie x_i, $0 \leq x_i \leq A$, $x_1 + x_2 = A$, and the date of agreement on both issues t, $0 \leq t \leq \infty$. The privately-known cost of party i for a global agreement is $c_i = c_{i1} + c_{i2}$, which is therefore drawn from the support $\left[\underline{c}_{i1} + \underline{c}_{i2}, \bar{c}_{i1} + \bar{c}_{i2}\right]$. Thus, payoffs from a global agreement are

$$e^{-rt} (x_i - c_i).$$

A classical example is the bargaining between a seller (party 1) and a buyer (party 2) over the sale price of two goods, among which there is neither complementarity nor substitution. The seller's private values s_k ($k = 1, 2$) for the objects are distributed in the corresponding intervals $\left[\underline{s}_k, \bar{s}_k\right]$, while the buyer's private values are distributed in the intervals $\left[\underline{b}_k, \bar{b}_k\right]$, respectively. Denoting by $A_k = \bar{b}_k - \underline{s}_k$ the maximal conceivable gains from trading good k, we can cast this example in the above framework by setting $c_{1k} = s_k - \underline{s}_k$ and $c_{2k} = \bar{b}_k - b_k$, where trade in the price p_k corresponds to a share $x_1 = p_k - \underline{s}_k$ to the seller and $x_2 = \bar{b}_k - p_k$ to the buyer.

In separate bargaining, the parties discuss the sale price of each good on its own, and the trade of each good can take place once bargaining over its price is concluded. In global bargaining, in contrast, the parties negotiate an overall price for the *bundle* of the two goods, for which the seller's reservation value is $s_1 + s_2$ and the buyer's reservation value is $b_1 + b_2$.

The parties' reservation values s_k, b_k (and hence the corresponding costs $c_{1k} = s_k - \underline{s}_k, c_{2k} = \bar{b}_k - b_k$) remain meaningful entities even under global bargaining: s_k (b_k) is the minimum (maximum) price for which the seller (buyer) would be willing to sell (buy) good k had bargaining been carried out separately. However, only the sums $s_1 + s_2, b_1 + b_2$ (and hence the sum of costs $c_{11} + c_{12}, c_{21} + c_{22}$) matter when bargaining globally.

The same comment applies in general. Under global bargaining, the two issues are not "fused" into one big inseparable issue, so the costs of sharing each of the pies remain meaningful. However, only the sums of the costs and the sum of the sizes of the two pies matter in the process of global bargaining. Thus, the model is suitable only for cases in which the two smaller problems can be solved as one and not just simultaneously. In global bargaining, a solution to the big problem is not given in terms of two solutions to the two smaller problems. Rather, there is one solution to the bundled problem.

3 The first negative effect of global bargaining: mixing good deals with bad ones

In what follows we will argue that separate bargaining may be preferred to global bargaining at the ex-ante stage—before the private costs are observed by the players. The reason for this result is simple: Under global bargaining, the gains to agreement are positive also when they are positive for one issue but negative (though smaller in absolute value) for the other. Under separate bargaining, a deal may be struck only for the former issue but not for the latter, yielding larger overall surplus. Although the magnitude of this effect over overall expected gains depends on the relative importance of each issue and the distribution of costs, it does arise under rather typical environments, such as in the following example:

Equally important issues and uniform costs. *Players $i = 1, 2$ can split two pies, two issues, of identical size, $A_k = 1/2$, $k = 1, 2$. The costs of executing each agreement on each issue, c_{ik}, is independently drawn with a uniform distribution on $[0, 1/2]$ for each i and k.*

In the associated global bargaining problem the players can split a pie of size $A_1 + A_2 = A = 1$, and for each i the costs of executing each global agreement $c_i = c_{i1} + c_{i2}$, is independently and identically distributed with density

$$f(c_i) = \begin{cases} 4c_i, & c_i \leq 1/2, \\ 4(1 - c_i), & c_i > 1/2, \end{cases}$$

on $[0, 1]$; so that the potential ex-ante surplus attainable in global bargaining is

$$\int_0^1 \int_0^{1-c_1} (1 - c_1 - c_2) f(c_2)\, \mathrm{d}c_2 f(c_1)\, \mathrm{d}c_1 = \frac{7}{60}.$$

Global or separate bargaining proceed according to the protocol of some Bayesian game, and the outcomes are the result of equilibrium play.

Assume that the bargaining games used to negotiate separately A_1 and A_2 are such that, in equilibrium, the optimal mechanism of Myerson and Satterthwaite (1983)[4] is implemented. The outcome induced by this mechanism is an immediate agreement for bargaining pairs with costs that satisfy $(1 - c_{1k} - c_{2k}) \geq 1/8$, and perpetual disagreement for all other pairs. Thus, the expected surplus for each issue is

$$\int_0^{3/8} \int_0^{3/8-c_1} (1 - c_1 - c_2)\, 2dc_2 2dc_1 = \frac{9}{128}.$$

Observe that $2 \times \frac{9}{128} > \frac{7}{60}$. Hence the benefits anticipated from separate bargaining of A_1 and A_2 are strictly greater that the potential gains over A. Now, since at *any* Bayesian bargaining game, the expected gains in equilibrium are always strictly below the first-best, we conclude that separate bargaining delivers expected gains that are strictly greater than those of global bargaining. The optimal performance of the mechanism implemented for separate negotiations is a sufficient condition for the strict superiority of separate bargaining, but of course it is not necessary. For example, separate bargaining remains strictly superior to global bargaining under a protocol where each party makes a take-it-or-leave-it offer on one of the issues: It is easily checked that this protocol delivers expected gains $2 \times \frac{1}{16} > \frac{7}{60}$.

4 The second negative effect of global bargaining: inefficiencies "add up"

Next, we establish that separate bargaining can be superior to global bargaining, not only in terms of ex-ante expected benefits, but also *ex post*—when the costs of each issue for both players are known. This holds whether or not the gains from agreement for one of the items turn out to be non-existent or negative. To make this claim we postulate a specific bargaining protocol—the game introduced by Wang (2000)—and demonstrate that, when players follow this protocol both for separate and global negotiations, the former deliver superior payoffs to *all pairs of bargainers, under every realization of the costs.* The rules of the game are as follows:[5]

The wang game. *Bargaining takes place in continuous time* $t \in [0, \infty)$. *Each party must decide if and when to opt out of the negotiation process, or otherwise if and when to approach the negotiation table. If at some point in time t one of*

[4] Myerson and Satterthwaite (1983) establish that first-best expected gains and equilibrium are incompatible, and they characterize the direct revelation mechanism that maximizes surplus subject to equilibrium constraints (i.e., Bayesian incentive compatibility and individual rationality). This mechanisms can be (indirectly) implemented by a sealed-bid double auction.

[5] See also footnote 2 above.

the parties has approached the table, the other party should decide if and when to make an offer regarding the split of the pie. Once an offer has been made by some party, the other party should decide whether to accept it, in which case it is implemented immediately. Otherwise, the party who received the offer should decide if and when to make a counter-offer, and she can do so whenever she likes after a minimal elapse t_0 *has passed. This alternating-offers scheme is then repeated until some offer is accepted, or otherwise indefinitely.*

In addition to tractability, the present game has important advantages for a positive analysis. First, its unique symmetric equilibrium is renegotiation-proof, in the sense that all the potential gains from agreement are eventually consummated. This is important, since a positive analysis would not be complete if it stops short of the stage in which both parties have no further incentives to continue the negotiations. Second, the equilibrium strategies depend only on the supports of the bargainers' distributions of valuations, and are therefore rather robust to higher-order uncertainties about the detailed specifications of these distributions. Robustness to higher-order uncertainties is important, given the ample experimental evidence that individuals often fail to carry out the reasoning that such considerations entail. Last, the resulting bargaining mechanism is in fact the unique undominated dominant incentive compatible dynamic mechanism that allows an agreement with an even split of the net surplus between all compatible types (Copic and Ponsati 2006).

The bargaining outcome. *The Wang game has a unique symmetric equilibrium in which the timing of the parties' moves completely reveal their cost types.*[6] *In the limit as* $t_0 \to 0$, *its outcome is easily described:*

A compatible bargaining pair, i.e., $c_1 + c_2 < A$, *agrees at shares*

$$x_i(c_1, c_2) = c_i + \frac{A - c_1 - c_2}{2}$$

at date

$$T(c_1, c_2) = -\frac{1}{r} \ln\left(\frac{A - c_i - c_j}{G}\right).$$

Incompatible pairs, i.e. $c_1 + c_2 \geq A$, *remain in disagreement.*

See Wang (2000) for a detailed proof. Here we give an heuristic discussion. The type of party i that approaches the table at time t is given by

$$C_i(t) = \underline{c}_i + \frac{1}{2}\left(1 - e^{-rt}\right) G.$$

[6] There is also a continuum of equilibria in which these moving times are asymmetric across the parties, but in all of them the eventual time to agreement is identical (Wang 2000). Hence, our analysis below is not sensitive to any choice among these equilibria.

That is, type $c_i \in \left[\underline{c}_i, \underline{c}_i + \frac{1}{2}G\right)$ approaches the table at the time

$$\tau_i\left(c_i; \underline{c}_i, G\right) = -\frac{1}{r}\ln\left(1 - \frac{2\left(c_i - \underline{c}_i\right)}{G}\right).$$

Accordingly, type $c_i \in \left(\underline{c}_i + \frac{1}{2}G, \underline{c}_i + G\right]$ opts out at the time $\tau^j\left(A - c_i; \underline{c}_j, G\right)$, because if by that time the other party j hasn't approached the negotiation table, this means that the sum of the costs $c_1 + c_2$ exceeds the worth A of the pie. Furthermore, any type $c_i > \underline{c}_i + G$ (if there exists such a type, i.e. if $\bar{c}_i > \underline{c}_i + G$) opts out immediately.

If party i has approached the negotiation table at the time t which corresponds to the type $c_i = C_i(t)$, it becomes common knowledge that the type c_j of the other party j satisfies $c_j \geq C_j(t)$ and hence that the net gains to agreement are at most

$$A - C_i(t) - C_j(t) = e^{-rt}G.$$

On the equilibrium path, the sub-game that follows is therefore a game with one-sided asymmetric information, about the type c_j of party j. Type c_j signals her type by delaying her offer further by

$$\mathcal{T}\left(c_i, c_j, G_t\right) = -\frac{1}{r}\ln\left(\frac{A - c_i - c_j}{e^{-rt}G}\right) = -\frac{1}{r}\ln\left(\frac{A - c_i - c_j}{G}\right) - t$$

Eventually, at the time

$$T\left(c_i, c_j\right) = t + \mathcal{T}\left(c_i, c_j, G_t\right) = -\frac{1}{r}\ln\left(\frac{A - c_i - c_j}{G}\right)$$

c_j proposes the Rubinstein (1982) split of the net gains $A - c_i - c_j$ in which she gets the share

$$c_j + \frac{A - c_i - c_j}{1 + \delta}$$

where $\delta = e^{-rt_0}$ is the discounting associated with the minimal elapse t_0 between offers.

In the ensuing subgame, the parties use their subgame-perfect equilibrium strategies in the Rubinstein (1982) game, and make their counter-offers as soon as they can (that is, t_0 after they receive an unacceptable offer). In particular, at equilibrium party i accepts immediately the above split proposal of j, and it is implemented immediately.

Restricting attention to the limiting case in which this elapse vanishes, i.e. $t_0 \to 0$, and hence $\delta \to 1$, the eventual net gain of each of the parties is given by

$$\frac{A - c_i - c_j}{2}.$$

The same agreement terms and timing can be obtained in a dynamic mechanism in which the parties disclose their cost types c_i, c_j to a mediator, who then implements this split after the delay $T\left(c_i, c_j\right)$. This mechanism is (1) ex-post individually rational, (2) ex-post incentive compatible, and (3) it assures that any gains to agreement are eventually materialized. Moreover, this mechanism is the unique Pareto dominant dynamic mechanism among all the dynamic mechanisms with properties (1)–(3) that splits the net surplus evenly (Copic and Ponsati 2006).

Since the discounting caused by the delay $T\left(c_i, c_j\right)$ is $\frac{A-c_i-c_j}{G}$, the benefits of a bargainer with cost c_i that faces a partner with cost c_j are easily described as a function of the net surplus:

The bargaining gains. *Let $\gamma = A - c_i - c_j$ denote the eventual net surplus for a bargaining pair. The discounted net gain of each of the parties is*

$$g\left(\gamma, G\right) = \begin{cases} \frac{\gamma}{G} \times \frac{\gamma}{2} = \frac{\gamma^2}{2G}, & \gamma > 0, \\ 0, & \gamma \leq 0. \end{cases}$$

Let us now consider our original set up, in which there are two pies to share with positive sizes A_1, A_2. Recall that by assumption $\underline{c}_{1k} + \underline{c}_{2k} < A_k$ for $k = 1, 2$. Denote by

$$G_k = A_k - \underline{c}_{1k} - \underline{c}_{2k} > 0$$

the maximal conceivable net gains to agreement on pie k and by

$$\gamma_k = A_k - c_{1k} - c_{2k}$$

the realization of the net gains to agreement on pie k.

We may now compare the gains of separate bargaining with those of global bargaining:

Proposition 1 *For any realization of private costs*

$$\left(c_{11}, c_{12}, c_{21}, c_{22}\right) \in \left[\underline{c}_{11}, \bar{c}_{11}\right] \times \left[\underline{c}_{12}, \bar{c}_{12}\right] \times \left[\underline{c}_{21}, \bar{c}_{21}\right] \times \left[\underline{c}_{22}, \bar{c}_{22}\right]$$

separate bargaining yields larger discounted net gains than does global bargaining:

$$g\left(\gamma_1, G_1\right) + g\left(\gamma_2, G_2\right) \geq g\left(\gamma_1 + \gamma_2, G_1 + G_2\right)$$

and equality obtains only in the knife-edge cases in which

$$\frac{\gamma_1}{G_1} = \frac{\gamma_2}{G_2} > 0.$$

Proof There are three relevant cases.

1. $\gamma_1 > 0, \gamma_2 > 0$. In this case we have to show that

$$\frac{\gamma_1^2}{2G_1} + \frac{\gamma_2^2}{2G_2} \geq \frac{(\gamma_1 + \gamma_2)^2}{2\,(G_1 + G_2)}.$$

   Indeed, $\left(\frac{\gamma_1^2}{2G_1} + \frac{\gamma_2^2}{2G_2}\right) - \frac{(\gamma_1+\gamma_2)^2}{2(G_1+G_2)} = \frac{(\gamma_1 G_2 - \gamma_2 G_1)^2}{2G_1 G_2 (G_1+G_2)} \geq 0$ and equality obtains only in the knife-edge realizations in which $\frac{\gamma_1}{G_1} = \frac{\gamma_2}{G_2}$. Separate bargaining is superior to global bargaining in this case because the net surplus is eventually shared in the same way under both agendas, but under global bargaining the delay is longer and the discounting is heavier – the bargaining inefficiencies over the pies "add up."
2. $\gamma_1 > 0, \gamma_2 \leq 0, \gamma_1 + \gamma_2 > 0$. In this case separate bargaining yields a net discounted gain of $\frac{\gamma_1^2}{2G_1}$ for each party from the first pie and nothing from the second pie. Global bargaining creates a net discounted gain of $\frac{(\gamma_1+\gamma_2)^2}{2(G_1+G_2)}$ for each side. We have

$$\begin{aligned}\frac{\gamma_1^2}{2G_1} - \frac{(\gamma_1 + \gamma_2)^2}{2\,(G_1 + G_2)} &= \frac{\gamma_1^2 G_2 - G_1 \gamma_2\,(2\gamma_1 + \gamma_2)}{2G_1\,(G_1 + G_2)} \\ &\geq \frac{\gamma_1^2 G_2 - G_1 \gamma_2\,(\gamma_1 + \gamma_2)}{2G_1\,(G_1 + G_2)} > 0\end{aligned}$$

   The first inequality follows from the fact that $\gamma_2 \leq 0$ and $\gamma_1 > 0$. The second inequality is due to the fact that $\gamma_2 \leq 0$ and $\gamma_1 + \gamma_2 > 0$. In this case global bargaining combines a good deal on the first pie with a bad deal on the second, while the latter is avoided in separate bargaining.
3. $\gamma_1 \leq 0, \gamma_2 \leq 0$, and therefore $\gamma_1 + \gamma_2 \leq 0$. In this case there is no agreement whatsoever under either agenda. □

Remark 1 The conclusion of Proposition 1 would not be maintained if we were to assume that even under separate bargaining, the deal is implemented only once both issues are resolved. To see this, assume, without loss of generality that $\frac{\gamma_1}{G_1} > \frac{\gamma_2}{G_2}$ and that $\frac{\gamma_1}{G_1} > 0$, so that issue 1 is resolved first in finite time. The implementation of the agreement for issue 1 would then be postponed until issue 2 is resolved. Here there are two main cases:

(a) If $\gamma_2 > 0$, then the the discounting applied to both issues will be $\frac{\gamma_2}{G_2}$. Thus, the gains of each party from separate bargaining would be

$$\frac{\gamma_2}{G_2}\left(\frac{\gamma_1}{2}+\frac{\gamma_2}{2}\right)$$

which are smaller than the gains to global bargaining

$$\frac{(\gamma_1+\gamma_2)^2}{2\,(G_1+G_2)}$$

This is because

$$\frac{\gamma_2}{G_2}\left(\frac{\gamma_1}{2}+\frac{\gamma_2}{2}\right)-\frac{(\gamma_1+\gamma_2)^2}{2\,(G_1+G_2)}=\frac{(\gamma_1+\gamma_2)\,G_1}{2\,(G_1+G_2)}\left(\frac{\gamma_2}{G_2}-\frac{\gamma_1}{G_1}\right)<0$$

due to our assumption that $\frac{\gamma_1}{G_1} > \frac{\gamma_2}{G_2}$.

(b) If $\gamma_2 \leq 0$, then issue 2 would never be resolved with an agreement (though the fact that there are no gains to agreement on this issue would be realized in finite time, when one of the parties opts out[7]). Thus, in case 2 above in which $\gamma_1 + \gamma_2 > 0$, global bargaining would be better than this version of separate bargaining, because global bargaining would lead to a mutually-beneficial package deal in finite time (though this package deal would mix a good deal for issue 1 with a bad deal for issue 2).

Is the alternative assumption in this remark reasonable? For instance, is it reasonable for the parties not to implement the agreement for issue 1 even when they realize that there are no gains to agreement on issue 2? If the answer is positive, this means there is a strong linkage between the two issues in the first place, so *bargaining separately is not an effective option* to start with. The reason is that even if negotiation on the two issues is carried out concurrently in two distinct rooms, it is clear from the outset that any agreement reached in one room can only become effective if and when agreement is reached also in the other room.

5 Endogenous agenda

In the previous section we assumed that under separate bargaining, the split of each pie is negotiated in isolation from the split of the other, with no communication between representatives who discuss the different pies. In this section we show that this Chinese Wall is unnecessary. Even if the negotiation is carried out at a unique location, and the sides can decide whether to move or make

[7] Except for the knife-edge case in which $c_{12} = c_{22} = \frac{G}{2}$ (and so $\gamma_2 = 0$). In this very particular case, both parties never approach the negotiaion table but never opt out.

offers on a single pie or on the combined bundle, at equilibrium they will choose to bargain each pie separately.

Specifically, allow each party to decide if and when to opt out of the negotiations on either one of the issues or both of them simultaneously, or otherwise if and when to approach the negotiation table for one of the issues or the global bargaining table for the combined pie. Hence we call this bargaining protocol *the endogenous agenda protocol.* The rules of this protocol are as follows.

The endogenous agenda protocol. *If some party opts out from negotiating both issues, bargaining ends without sharing the pies. If some party opts out negotiating one of the issues, bargaining continues only on the remaining issue, so the global bargaining table cannot be approached any more. If some party approaches the bargaining table for one of the issues, negotiation will proceed separately for each of the issues, with the same rules as in the Wang game for each of them, and the global bargaining table cannot be approached any more. From that point on, we say that the negotiations proceed with the separate bargaining agenda. Conversely, if one of the parties approaches the global bargaining table, negotiation will proceed for the combined pie according to the rules of the Wang game, and the individual-issue tables cannot be approached any more. From that point on, we say that the negotiations proceed with the global bargaining agenda. Finally, if one of the parties approaches the global bargaining table, while the other party approaches simultaneously a single-issue table, the choice of the agenda for the sequel is determined by the flip of a fair coin.*

We will consider two scenarios. We call the first scenario *separate bargaining on the equilibrium path with global bargaining off the equilibrium path.* We call the second scenario *global bargaining on the equilibrium path with separate bargaining off the equilibrium path.*

In the first scenario, the players expect each other to follow the Wang equilibrium separately for each issue, and to ignore the option to approach the global bargaining table. If one of them nevertheless deviates and does so, the other player believes that the deviation occurred at the equilibrium time of the Wang game in which only global bargaining is feasible. That other player then follows her equilibrium part in the sub-game that follows, given that belief and the information revealed about her costs by her equilibrium behavior (in the separate bargaining agenda) up till that point. The deviator then responds by his own equilibrium behavior in this sub-game given this belief and information, so that the behavior in the sub-game is a sequential equilibrium.

Things are exactly reversed in the second scenario. Here, the players expect each other to follow the equilibrium of the global bargaining agenda, and to ignore the option to approach any of the separate bargaining tables. If one of them deviates and does approach one of them, the other player believes that the deviator followed the approaching timing in the equilibria of the Wang games, separately for each of the pies. That other player then follows her part in the equilibria of the Wang games for the pies, given that belief and the information revealed about her costs by her equilibrium behavior (in the global bargaining agenda) before the deviation. The deviator then responds by his own equilibrium behavior, separately for each of the pies, given that belief and information.[8]

We now turn to show that the first scenario constitutes a sequential equilibrium, while the second scenario does not.

Proposition 2 *In the endogenous agenda protocol*: (i) *there exists a sequential equilibrium with separate bargaining on the equilibrium path and global bargaining off the equilibrium path*; *and* (ii) *there does not exist a sequential equilibrium with global bargaining on the equilibrium path and separate bargaining off the equilibrium path.*

Proof We first establish (i). Consider an (alleged) equilibrium strategy profile satisfying the conditions of the first scenario, suppose that party i deviates and approaches the global bargaining table at time t. By our assumption, the other party j then believes that i has done so at the equilibrium time of the global bargaining game, and hence that the sum of the costs of i is

$$C_i(t) = \underline{c}_{i1} + \underline{c}_{i2} + \frac{1}{2}\left(1 - e^{-rt}\right)(G_1 + G_2).$$

Now, since j followed her equilibrium behavior in the separate bargaining equilibrium up to t, it is common knowledge that her costs for the pies $k = 1, 2$ are at least

$$C_{jk}(t) = \underline{c}_{jk} + \frac{1}{2}\left(1 - e^{-rt}\right)G_k$$

and hence that the sum of these costs is at least

$$C_j(t) = \underline{c}_{j1} + \underline{c}_{j2} + \frac{1}{2}\left(1 - e^{-rt}\right)(G_1 + G_2).$$

This is exactly the information that would be revealed about player j at time t if bargaining were confined to global bargaining in the first place, and hence after the deviation the game will proceed as in the Wang equilibrium for global bargaining. By Proposition 1 above the resulting payoff for the deviator would be no larger than without the deviation.

It remains to verify that on the equilibrium path, when the global bargaining table is ignored, the combination of the Wang equilibrium strategies for each player is indeed a best reply to the other player's combination. As time goes by, observing the other player's moves (or their absence) regarding one issue may lead a player to update her beliefs about the other player's cost in the other issue *differently* than she would if she were unable to observe those moves, because the other player's costs may be *correlated* across the issues. However, this difference does not pertain to the *supports* of the costs attributed to the

8 In the proof of Proposition 2 below we will show that this continuation after the deviation is a sequential equilibrium.

other player in the two issues. This follows from the assumption that the support of the costs c_{ik} is the product of rectangles, and from the fact that the strategies of the Wang equilibrium, which satisfy sequential rationality, depend only on the relevant support of the costs in each sub-game and not on the particular posterior belief on this support.

We thus conclude that the behavior in the first scenario is a sequential equilibrium, and the proof of (i) is completed.

Next we establish (ii). Consider an (alleged) equilibrium strategy profile satisfying the conditions of the second scenario, suppose that i deviates and approaches one of the separate bargaining tables – that of pie 1, say—at time $t > 0$. By assumption, the other party j then believes that the cost of i in pie 1 is

$$C_{i1}(t) = \underline{c}_{i1} + \frac{1}{2}\left(1 - e^{-rt}\right) G_1$$

Since j followed the global bargaining equilibrium prior to the deviation time t, it is common knowledge that the sum of her costs $c_{j1} + c_{j2}$ is at least

$$C_j(t) = \underline{c}_{j1} + \underline{c}_{j2} + \frac{1}{2}\left(1 - e^{-rt}\right)(G_1 + G_2).$$

Hence, it is common knowledge that for each individual pie $k = 1, 2$, the cost c_{jk} of j is at least

$$\widetilde{C}_{jk}(t) = \max\left\{C_j(t) - \overline{c}_{j(2-k)}, \underline{c}_{jk}\right\}$$

(because the sum of the costs $C_j(t)$ could potentially be mostly due to the other pie $2 - k$ rather than to pie k). This lower bound $\widetilde{C}_{jk}(t)$ on the cost c_{jk} is smaller than the lower bound

$$C_{jk}(t) = \underline{c}_{jk} + \frac{1}{2}\left(1 - e^{-rt}\right) G_k$$

on c_{jk} if issue k were negotiated separately –

$$\widetilde{C}_{jk}(t) < C_{jk}(t),$$

because both

$$\underline{c}_{jk} - C_{jk}(t) < 0$$

and

$$\begin{aligned}\left(C_j(t) - \overline{c}_{j(2-k)}\right) - C_{jk}(t) &= \left(\underline{c}_{j(2-k)} + \frac{1}{2}\left(1 - e^{-rt}\right) G_{(2-k)}\right) - \overline{c}_{j(2-k)} \\ &= C_{j(2-k)}(t) - \overline{c}_{j(2-k)} < 0.\end{aligned}$$

We will now consider in turn each of the pies.

By our assumption, from time t and on pie 1 will be negotiated according to the Wang equilibrium for pie 1 in the sub-game in which it is common knowledge that i's cost is $C_{i1}(t)$, but j's cost c_{j1} is known to be in the interval $\left[\widetilde{C}_{j1}(t), \overline{c}_{j1}\right]$ rather than in the interval $\left[C_{j1}(t), \overline{c}_{j1}\right]$. This means that a type of player j with cost c_{j1} will offer to split the net gains for pie 1 at the time

$$t + \left(-\frac{1}{r}\ln\left(\frac{A_1 - c_{i1} - c_{j1}}{A_1 - C_{i1}(t) - \widetilde{C}_{j1}(t)}\right)\right),$$

sooner than at the time to agreement

$$t + \left(-\frac{1}{r}\ln\left(\frac{A_1 - c_{i1} - c_{j1}}{A_1 - C_{i1}(t) - C_{j1}(t)}\right)\right) = -\frac{1}{r}\ln\left(\frac{A_1 - c_{i1} - c_{j1}}{G_1}\right)$$

if pie 1 were negotiated separately in the first place.

We now turn to pie 2. We assumed that if i deviates at time t, the other player j believes that i's cost c_{i2} of pie 2 is at least $C_{i2}(t)$, i.e.,

$$c_{i2} \in \left[C_{i2}(t), \overline{c}_{i2}\right].$$

Furthermore, since player j followed the global bargaining equilibrium, it is common knowledge that j's cost c_{j2} is in the interval $\left[\widetilde{C}_{j2}(t), \overline{c}_{j2}\right]$. Thus, from time t and on the players will engage in the Wang equilibrium for pie 2 with these intervals, and split the net gains for it at the time

$$t + \left(-\frac{1}{r}\ln\left(\frac{A_2 - c_{i2} - c_{j2}}{A_2 - C_{i2}(t) - \widetilde{C}_{j2}(t)}\right)\right),$$

sooner than at the time to agreement

$$t + \left(-\frac{1}{r}\ln\left(\frac{A_2 - c_{i2} - c_{j2}}{A_2 - C_{i2}(t) - C_{j2}(t)}\right)\right) = -\frac{1}{r}\ln\left(\frac{A_1 - c_{i1} - c_{j1}}{G_1}\right)$$

if pie 2 were negotiated separately in the first place.

In the entire sub-game which follows the deviation, where both pies are negotiated in parallel, the above strategies form a sequential equilibrium. As explained above for the first scenario, this follows from the fact that the supports of the beliefs about the costs in one issue evolve with time in the same way, whether or not the players can observe the course of bargaining on the other issue.

We conclude that after i's deviation that imposes separate bargaining, the time to agreement on each of the issues could only be shorter relative a situation in which the issues were bargained separately in the first place, and hence the (equally-shared) total gains to agreement would be larger. By Proposition 1, they would be thus larger still than the gains from global bargaining. It follows that i's deviation at the time

$$t = \min_{k=1,2} \tau_i \left(c_{ik}; \underline{c}_{ik}, G_k\right) > 0$$

would be profitable for him.

Finally, it would also be profitable for player i to deviate at $t = 0$ if

$$t = \min_{k=1,2} \tau_i \left(c_{ik}; \underline{c}_{ik}, G_k\right) = 0,$$

i.e., if $c_{i1} = \underline{c}_{i1}$ or $c_{i2} = \underline{c}_{i2}$. Indeed, by proposition 1 the extra gains from separate bargaining is zero only in a set of knife-edge cases, to which player i assigns probability smaller than 1.

We thus conclude that the alleged strategy does not constitute an equilibrium. This competes the proof of (ii). □

Remark 2 The order in which the issues $k = 1, 2$ are settled at equilibrium depends not on the absolute gains to agreement γ_k, but rather on the ratio $\frac{\gamma_k}{G_k}$ between these gains γ_k and the maximal conceivable gains G_k from issue k. The larger this ratio is, the sooner is the issue resolved.

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