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BARGAINING AT VARIABLE RHYTHMS(*)

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Abstract This note presents a modification of Rubinstein's model in which players can choose whether to be fast or slow in responding to their opponent's proposals. We characterize the effects of such choice on the outcome of the bargaining and give predictions on what rhythm will players choose in equilibrium.

REGATEO A RITMOS VARIABLES

Resumen Esta nota presenta una modificación del modelo tradicional de Rubinstein en el que dos jugadores pueden escoger el ritmo al que responder las propuestas de sus rivales. Caracterizamos los efectos de esta elección sobre el resultado de la negociación y ofrecemos predicciones acerca del ritmo que los jugadores utilizarán en equilibrio.

1. INTRODUCTION

The standard analysis of negotiation processes viewed as non-cooperative bargaining situations is by now well developed from Rubinstein's (1982) seminal paper. Excellent reviews of this literature are found in Osborne and Rubinstein (1990) and Binmore and Dasgupta (1987). One of the characteristics of this analysis is the fact that players involved in the bargaining process are not able to choose the rhythm at which the negotiation develops, that is to say the frequency at which proposals and counterproposals are announced.

It is a common observation that negotiations are very seldom smooth processes. Sometimes, players agree to interrupt the process and restart it somer time later; some other times negotiations are broken by one of the parties unilaterally. A way to look at these interruptions is as an effort by the players that provoke the interruption to affect the initial status quo. We feel that this attitude is a crucial element to obtain a proper understanding of the bargaining process. In other words, we believe that the endogeneization of the rhythm at which bargaining takes place will provide insights not available in the standard Rubinstein's model.

We propose to consider these types of behavior as a change in the rhythm of bargaining. Accordingly, we will present a model in the line of Rubinstein's where players are able to choose the rhythm of offers and counter-offers. In particular, players will be able to choose whether to be fast or slow in responding to their opponent's proposals. We characterize the effects of such choice on the outcome of the bargaining and give predictions on what rhythm will players choose in equilibrium.

Similar issues are addressed by Sákovics (1993) and Perry and Reny (1993). In their models, bargaining takes place in continuous time and players can make offers at any

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time without any predetermined order, with a fixed minimum delay between offers of the same players and between offers of different players. If the players can react immediately to their opponent's offers, then the set of equilibria approaches the Rubinstein outcome. If it takes time to react then there are multiple equilibria that yield very different partitions of the surplus, some of them involving substantial delay.

Since the order of players' offers as well as the possible rhythms are fixed in our model, our extension is not as rich as the one of Sákovics and Perry and Reny. However, since our simple extensive form yields a unique (subgame perfect) equilibrium, we can characterize the equilibrium outcome for each configuration of the parameters. Thus, we can explicitely relate the abilities to change rhythms and the possible delays under each rhythm to the share of the surplus that each player obtains. Our most salient result is that, unlike in the Rubinstein model, the first mover advantage does not vanish as the bargaining becomes frictionless, actually, the first player to make an offer obtains all the surplus.

2. THE MODEL

Consider the following bargaining game. Two players A and B take turns to make offers on how to split a cake of size one. An outcome $(x, 1-x, t)$ assigning a portion x to player A and $1-x$ to player B at date t is reached if proposal x is accepted at date t. Players preferences over pairs (x, t), a portion of the pie and a date of agreement, are represented by utility functions $u^i(x, t) = xe^{-\eta t}$, $i = A, B$, $\eta = a, b$. We can also write $u^i(x, t) = x\delta^{im}$ with $\delta^i = e^{-\eta\Delta}$, where $t = m\Delta$, and Δ is the real time interval between proposals.

There may be k, $(k = 1, 2, \ldots)$ different pairs of rythms at which proposals can be exchanged. Given a pair of rythms (Δ_k^i, Δ_k^j), where $i = A$ $(j = B)$ we denote by R_k the pair of discounts that fully characterizes it. Abusing terminology R_k will be called a rule. At $t = 0$ nature decides who plays first (i.e. who is called player A) and an initial rule. After hearing an offer, player i can accept it, and the game ends or reject it. Should he reject an offer he can take two decisions: either make a counteroffer within the same rule or make a new offer in the context of an alternative rule accelerating or delaying next proposal (i.e. change the rhythm of the bargaining process). We assume that changes of a rule take effect immediately. We have also considered the case when the change of rules takes one extra period, i.e. players are forced to respond in the current rhythm but they can change the rhythm at which they will hear their opponent's response. The results in this case are qualitatively very similar and the analysis is analogous.

As Rubinstein (1982) showed, a game with fixed rule $R_k = (\delta_k^A, \delta_k^B)$ has a unique Perfect Equilibrium Partition (PEP) in which player A receives $\dfrac{1-\delta_k^B}{1-\delta_k^A\,\delta_k^B}$ and player B receives $\dfrac{\delta_k^B(1-\delta_k^A)}{1-\delta_k^A\,\delta_k^B}$.

Definition 1. *We say that rule R_m is effective at rule R_k iff player B prefers to delay agreement one period and obtain the PEP of the fixed rule game with R_m rather than obtaining the PEP of the fixed rule game with R_k.*

For convenience, we will develop our analysis in terms of the real time intervals betwen proposals. If there are only two possible rhythms of bargaining Δ and Δ', every player has two discount rates: a default rate $e^{-\eta\Delta}$ and a discount rate faced by the player that performs a switch of rules, $e^{-\eta\Delta'}$. In this set-up there are four possible sets of rules, $R_1=(e^{-a\Delta}, e^{-b\Delta})$; $R_2=(e^{-a\Delta}, e^{-b\Delta'})$; $R_3=(e^{-a\Delta'}, e^{-b\Delta'})$; $R_4=(e^{-a\Delta'}, e^{-b\Delta})$.

Lemma 1. *If we are at a rule R_k such that rule R_{k+1} is not effective, the PEP of the fixed rule R_k prevails.*

Proof: First notice that no player has any interest ever to change from rule R_k to rule R_{k+1}, to rule R_{k+2}, to rule R_{k+3}, and back to rule R_k. Therefore there are four possible situations:

(1) The PEP of the fixed rule R_k prevails.

(2) Player j does not want the outcome of the fixed rule R_{k+1}, but he uses his ability to change to R_{k+1}, because it can lead to rule R_{k+2}. This will be the case if player i, being at rule R_{k+1} is willing to change to the fixed rule R_{k+2}. But if player i prefers the outcome of the fixed rule R_{k+2} to the outcome of the fixed rule R_{k+1}, then player j has to prefer the outcome of the fixed rule R_{k+1} to the outcome of the fixed rule R_{k+2}. Hence, for player j either the outcome of the fixed rule R_k is better than the ones associated to R_{k+1} and to R_{k+2}, or he will not use his ability to change to rules R_{k+1} in order to get to R_{k+2}.

(3) Player j does not want the outcome of the fixed rule R_{k+1}, neither wants to use his ability to change to R_{k+1} in order to get to R_{k+2}, but to use it to arrive to R_{k+3}. To arrive to R_{k+3} requires that player i will want to change to R_{k+2} to reach R_{k+3}. But this is not possible by situation 2 above.

(4) Player j does not want the outcome of the fixed rule R_{k+1}, neither wants to use his ability to change to R_{k+1} in order to get to R_{k+2}, but to use it to arrive to $R_{k+4}=R_k$. This is clearly not possible by a similar argument as in the previous situation.

Lemma 1 tells us that given an initial rule $R_0=R_k$, player B may change the rhythm to R_{k+1} if k is odd, or to R_{k-1} if k is even. Accordingly, we need to check

- R_4 will be effective at R_3 iff

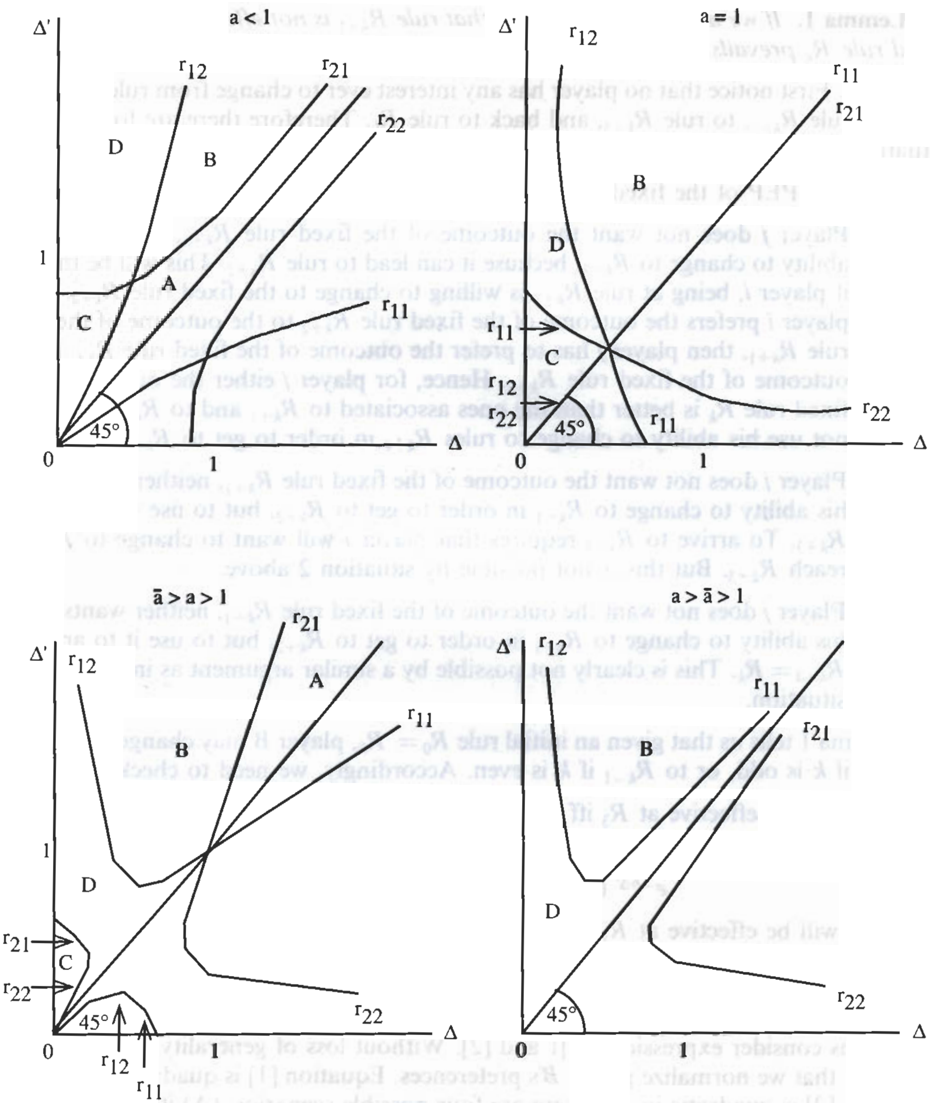
$$\frac{e^{-b\Delta}(1-e^{-b\Delta})}{e^{-b\Delta'}(1-e^{-a\Delta'})} > \frac{1-e^{-a\Delta'}e^{-b\Delta}}{1-e^{-(a+b)\Delta'}} \qquad [1]$$

- R_2 will be effective at R_1 iff

$$\frac{e^{-b\Delta'}(1-e^{-b\Delta'})}{e^{-b\Delta}(1-e^{-a\Delta})} > \frac{1-e^{-a\Delta}e^{-b\Delta'}}{1-e^{-(a+b)\Delta}} \qquad [2]$$

Let us consider expressions [1]1 and [2]. Without loss of generality we can assume $b=1$, so that we normalize player B's preferences. Equation [1] is quadratic in Δ, while equation [2] is quadratic in Δ'. There are four possible scenarios; (A) if both equations hold, (B) if only [1] holds, (C) if only [2] holds, and (D) if neither equation holds. We can visualize the different scenarios depending on the value of a, by plotting the roots of [1] and [2] in the space (Δ, Δ'). These are ilustrated in figure 1, where r_{ij} stands for root j of equation i.

Figure 1



2.1 Equilibrium

Proposition 1. *Let $\Delta < \Delta'$ and $b = 1$.*

- *In scenario A the PEP of the fixed rule R_2 prevails if $a \geq 1$ or R_4 if $a < 1$.*
- *In scenario B the PEP of the fixed rule R_1 prevails for any value of a.*
- *In scenario C the PEP of the fixed rule R_2 prevails for any value of a.*
- *In scenario D the PEP of the fixed rule R_3 prevails for any value of a.*

To prove the proposition it is convenient to state the following lemma:

Lemma 2. *Let x_k^i denote the portion of the cake assigned to player i in the PEP of the fixed rule R_k.*

(i) If we are at a rule R_k such that rule R_{k+1} is effective and rule R_{k+2} is not effective at R_{k+1}, the unique PEP is:

$$1-\delta_{k+1}^B\, x_{k+1}^B;\quad \delta_{k+1}^B\, x_{k+1}^B \qquad [3]$$

(ii) If we are at a rule R_k such that rule R_{k+1} is effective and rule R_{k+2} is also effective and R_{k+3} is not effective at R_{k+2}, the unique PEP is:

$$1-\delta_{k+1}^B\,(1-\delta_{k+2}^A\, x_{k+2}^A);\quad \delta_{k+1}^B\,(1-\delta_{k+2}^A\, x_{k+2}^A) \qquad [4]$$

(iii) If we are at a rule R_k such that rule R_{k+1} is effective, rule R_{k+2} is effective, R_{k+3} is effective, and R_k is not effective at R_{k+3}, the unique PEP is:

$$1-\delta_{k+1}^B\,[1-\delta_{k+2}^A\,(1-\delta_{k+3}^B\, x_{k+3}^B)];\quad \delta_{k+1}^B\,[1-\delta_{k+2}^A\,(1-\delta_{k+3}^B\, x_{k+3}^B)] \qquad [5]$$

Proof: Statement *(i)* follows immediately from Lemma 1. Statement *(ii)* follows from lemma 1 and statement *(i)* in lemma 2. Statement *(iii)* follows from lemma 1 and statements *(i)* and *(ii)* in lemma 2.

Proof of proposition 1:
From lemma 2, player A chooses $R_0 = R_k$ such that R_{k+1} is not effective.

- Consider scenario *A*. Rules that have no effective alternative are R_2 and R_4. The unique PEP, in terms of the payment to player *A* are $\frac{1-e^{-\Delta'}}{1-e^{-a\Delta}\, e^{-\Delta'}}$ and $\frac{1-e^{-\Delta}}{1-e^{-a\Delta'}\, e^{-\Delta}}$ respectively. Player *A* prefers R_2 to R_4 if

$$\frac{1-e^{-\Delta'}}{1-e^{-a\Delta}\, e^{-\Delta'}} > \frac{1-e^{-\Delta}}{1-e^{-a\Delta'}\, e^{-\Delta}}.$$

  which holds iff $a \geq 1$.

- Consider scenario *B*. Rules that have no effective alternative are R_1 and R_4. The unique PEP, in terms of the payment to player *A* are $\frac{1-e^{-\Delta}}{1-e^{-a\Delta}\, e^{-\Delta}}$ and $\frac{1-e^{-\Delta}}{1-e^{-a\Delta'}\, e^{-\Delta}}$ respectively. Player *A* prefers R_1 to R_4 if

$$\frac{1-e^{-\Delta}}{1-e^{-a\Delta}\, e^{-\Delta}} > \frac{1-e^{-\Delta}}{1-e^{-a\Delta'}\, e^{-\Delta}}.$$

  which holds for any value of *a*.

- Consider scenario C. Rules that have no effective alternative are R_2 and R_3. The unique PEP, in terms of the payment to player A are $\frac{1-e^{-\Delta'}}{1-e^{-a\Delta}\,e^{-\Delta'}}$ and $\frac{1-e^{-\Delta'}}{1-e^{-(a+1)\Delta'}}$ respectively. Player A prefers R_2 to R_3 if

$$\frac{1-e^{-\Delta'}}{1-e^{-a\Delta}\,e^{-\Delta'}} > \frac{1-e^{-\Delta'}}{1-e^{-(a+1)\Delta'}}.$$

  This inequality holds for any value of a.
- Consider scenario D. Rules that have no effective alternative are R_1 and R_3. The unique PEP, in terms of the payment to player A are $\frac{1-e^{-\Delta}}{1-e^{-\Delta\,(a+1)}}$ and $\frac{1-e^{-\Delta'}}{1-e^{-\Delta'\,(a+1)}}$ respectively. Player A prefers R_3 to R_1 if

$$\frac{1-e^{-\Delta'}}{1-e^{-\Delta'\,(a+1)}} > \frac{1-e^{-\Delta}}{1-e^{-\Delta\,(a+1)}}.$$

  This inequality holds for any value of a.

Proposition 1 tells us that scenarios B and D have equilibrium rules in which both players follow the same rhythm. In these scenarios the possibility of changing rhythm has no effect and the surplus is divided as in the Rubinstein game.

Scenarios A and C have equilibrium rules where players use different rhythms. Of these the most interesting case is scenario C. A close study of it leads to the following.

Proposition 2. *Let $\Delta < \Delta'$ and, for simplicity, $a=b=1$. For each share of the surplus $\alpha < 1$, there is a rhythm of bargaining $\underline{\Delta}$ such that if $\Delta < \underline{\Delta}$, then the share of the surplus assigned to player A is greater than α.*

Proof: If $a=b$ we are in scenario C if the rhythms of bargaining are not too large.

Hence the portion assigned to player A is,

$$\frac{1-e^{-\Delta'}}{1-e^{-\Delta'}\,e^{-\Delta}}$$

Note that this portion is larger than α provided that

$$e^{-\Delta'} < \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-e^{-\Delta}}$$

which holds for Δ small enough.

Remark 1: As bargaining becomes frictionless, the player moving first captures all the surplus.

Remark 2: Giving players the ability to choose their own initial rythms, does not substantially affect the results. Player A chooses the rythm giving him the best PEP payoff anticipating B's choice of rythm.

Remark 3: Considering more than two rythms, does not change the results. Only the fastest and the slowest rythms matter.

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