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Compromise vs Capitulation in Bargaining with Incomplete Information

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ABSTRACT. – We analyse a two-sided incomplete information negotiation that can reach three possible settlements: either the extreme proposal of one of the parties prevails, or an intermediate compromise is reached. Our main results are (i) that the outcome of the game in which no compromise is possible is an equilibrium of the game with compromise and that (ii) there exist equilibria in which compromise is reached with positive probability that are (ex-ante) Pareto improving. The results support the perception that mediators, restricting direct communication and/or coordinating the timing of concessions, enhance the efficiency of bargaining.

Compromis versus capitulation dans les négociations avec information incomplète

RÉSUMÉ. – Ce papier analyse le processus de négociation sous information incomplète entre deux parties dont l'issue peut être, soit que la proposition extrême d'une des parties prévale, ou bien qu'un compromis intermédiaire soit accordé. Les principaux résultats sont les suivants : (i) la solution du jeu dans lequel le compromis n'est pas possible est un équilibre du jeu avec compromis, et (ii) il existe des équilibres, dans lesquels le compromis est atteint avec probabilité positive, qui sont (ex-ante) Pareto supérieurs. Les résultats obtenus supportent l'intuition que les médiateurs, en contraignant la communication directe et/ou la coordination dans la séquence des concessions, permettent d'augmenter l'efficacité du processus de négociation.

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1 Introduction

Settling a dispute, whether by a compromise or by one of the party's total concession, usually takes time. Sometimes, after bargaining for a long time, parties walk away convinced that an agreement is impossible. The model of two-person bargaining presented in this paper provides insight on the forces driving conflicts towards compromise, capitulation, or stalemate and on the timing of these outcomes.

The basic bargaining problem that we study is the following: two parties are engaged in a negotiation that takes place over time and can reach three possible settlements, either the extreme proposal of one of the parties prevails, or an intermediate compromise is reached. The model, formally presented in section 2, is roughly described as follows: It is assumed that players can choose one among there possible partitions of one unit of surplus in continuous time. At each moment in time players know what proposals have been made up to then and choose whether to remain firm or to make a proposal more favorable to their opponent. Player's payoffs from any agreement decrease over time and depend on reservation values that are private information. There is a continuum of possible reservation values. Along a Perfect Bayesian Equilibrium (PBE), as time goes by, players update their beliefs about the type of opponent they face and adjust their behavior accordingly, choosing a best response to their opponent's behavior. Under the assumption that the ex-ante probability distribution of reservation values have a positive density on a full support, that is, any type that can extract some positive surplus from some agreement is present in the negotiation, we provide a full characterization of the set of Perfect Bayesian Equilibria.

The model that we present can be taken as a stylized model of bargaining situations in which, in addition to the proposals of each party, an obvious compromise is available. These are the simplest bargaining situations that are not purely concession games (as models in which only one of two extreme proposals could prevail would be). Under this simple set up, the essence of the bargaining problem with incomplete information is captured: the fact that making a concession has two different types of costs – the direct cost of accepting a smaller portion of the surplus right away, and the cost that players will pay in the future by having revealed themselves as weaker than their opponent.

Our results build on the structure of the unique Bayesian Equilibria (BE) of wars of attrition with two-sided uncertainty on the ability of the opponent to yield: Agreements occur only with delay and, if proposals are very extreme, the result is very inefficient (a long negotiation that yields an agreement with very small probability).

Here, taking the natural nex step, we study games in which parties have the possibility of reaching a compromise and provide a full characterization of the set of PBE outcomes. It turns out that games with two extreme proposals and a compromise do not necessarily yield a better outcome than the war of attrition. In fact, strategies in which the parties ignore the possibility of compromising remain an equilibrium of the game. These profiles, that we call War of Attrition Equilibria (WAE) are the unique ones that can be supported if players use markov strategies (i.e. the actions of a player at some partial history depend only on the standing proposals and her beliefs at that partial history). Apart from WAE, we characterize and show existence of Compromise Equilibria (CE). Along a CE, a compromise is reached with positive probability. It is possible that, whether by a total concession of one of the parties or by a compromise, an agreement is reached after substantial delay. Moreover, it is also possible that, after long negotiation, the parties reach the conclusion that no agreement is possible.

A canonical example of the bargaining situation that we analyze would be the distribution of four units of an indivisible good among two individuals. There is, however, an interpretation of the model that gives it a more general appeal: if two parties are bargaining with the help of a mediator, it is often the case that the latter proposes a compromise settlement to both parties, and then each party accepts or rejects the compromise settlement.

Since our characterization of the set of PBE with three possible agreements gives a robust characterization of the basic qualitative features that arise in bargaining with two sided incomplete information, the model can also be seen as a step towards a model with any finite set of possible partitions of the surplus. However, such a generalization is not straightforward because the results with three partitions of the surplus rely heavily on the particularly simple structure of the equilibrium strategies in the war of attrition. For more general games, characterizing equilibrium strategies after some partial concession occurs has so far proven untractable.

If we interpret the three agreement game as a stylized model of bargaining with the help of a mediator, our results suggest that third parties should not only propose a compromise: they should also coordinate the timing of bargaining and serve as communication channels. If communication takes place through a mediator, it is likely that she can improve the efficiency of the outcome by preventing too much revelation from taking place when players decide to make a concession. Even if the mediator does nothing but coordinating the schedule of the negotiation, she can induce the players to choose a CE over the WAE. In cases where the proposals of the parties are very extreme, the WAE yields very low expected payoffs, thus playing a CE will be welfare improving. Hence, the results are suggestive of a rationale for the extended practice of mediation as a mechanism for conflict resolution.

The remainder of the paper is organized as follows. Section 2 motivates our model in relation to the literature in sequential bargaining and to the litterature on the war of attrition. Section 3 formally presents the model. Section 4 presents a characterization of BE for games with two possible partitions. In section 5 the results for games with three possible partitions are presented. Some conclusions follow.

2 Relation to the Literature

The efficient agreement without any delay predicted by the Rubinstein-Sthal model seems to be at odds with the perception that delays and impasses are commons in all kinds of negotiations. As a consequence much effort was made to show that, with private information equilibrium outcomes may turn out to be inefficient ex-post because some types of players are willing to wait in order to signal their strength. (See, OSBORNE and RUBINSTEIN [1990] and references therein). Nevertheless, GUL and SONNENSCHEIN [1988], point out that models of sequential bargaining with one-sided private information may not be a robust basis for the analysis of disagreement in bargaining. They prove that if strategies are stationary, then delays obtained in bargaining models with one-sided private information vanish as the game approaches a continuous time structure, and, in the limit, the informed party is able to obtain all the surplus without delay.

Most real life bargaining situations are best described as sequential games with two-sided private information. The equilibria of this class of games are, however, very difficult to characterize. There are immense problems related to information updating and clear-cut results have been elusive. (See, for example, CHO [1990] and AUSUBEL and DENECKERE [1993] for very valuable partial results).

In sharp contrast with the lack of general results characterizing sequential bargaining with two sided private information, models of the war of attrition with two-sided private information (KREPS and WILSON [1982], OSBORNE [1985], PONSATI and SÁKOVICS [1995a]) have unique BE in many instances. Models of bargaining with only two possible agreements are wars of attrition, thus uniqueness of the equilibrium under two-sided private information is obtained by CHATTERJEE and SAMUELSON [1987]. In these model there are only two types of players: weak types, i.e. players that can obtain a positive payoff from a concession, and tough types, i.e. players that can only obtain a negative payoff from a concession. In the unique equilibrium, provided that they are weak, both players distribute their concessions on the same interval of time so that they simultaneously reach the conclusion that their opponent is tough. A player more likely to be tough receives a higher expected payoff. Furthermore, reaching an agreement takes time because a strategy in which all weak types of a given player concede at the first instant cannot be supported as an equilibrium; waiting an instant to convince the opponent of their toughness always increases the expected payoff for weak players. Moreover, while the alternating offers extensive form is crucial for all the results in models that assume a continuum of possible divisions of the surplus (with or without incomplete information), the uniqueness of the BE in wars of attrition with two-sided incomplete information is robust to a variety of extensive forms: neither time being continuous or discrete, nor alternating or simulaneous moves extensive forms are crucial for the results.

In relation to the literature, this essay is an attempt to address the problem of sequential bargaining with incomplete information by using the techniques and results from the litterature on wars of attrition. Here the assumption that agreements must lie in a finite set plays a crucial role: players cannot change their proposals in a continuous way, either they keep their proposals of they forego some positive amount of surplus. The original motivation for the present research was the conjecture that representing a bargaining game as a many stage war of attrition would provide a tractable game form, yet one rich enough to yield substantive results about the problem of bargaining with two-sided private information. This turns out to be true if the set of agreements is limited to two extreme proposals and a compromise, for more general games things are still unclear.

Since the litterature usually presents bargaining as a discrete time alternating move game, time being continuous may look as a striking feature of the present model. However, continuous time is not essential for our results. What is essential is the *simultaneous move* character of the bargaining process and the *finiteness of possible divisions of the surplus*: the same qualitative results could be obtained with a discrete time simultaneous move extensive form, in the limit as the interval between periods becomes negligible.

The reasons why a continuous time structure is chosen are the following. First, a simultaneous move game is crucial to obtain the results when more than two agreements are possible. It is necessary because we want to allow the possibility of simultaneous concessions leading to a compromise that both players prefer to a total concession. Moreover, although the alternating move extensive form is crucial to obtain uniqueness of the equilibrium in the Rubinstein model (see STAHL II [1990]), it is of no special benefit with a finite number of divisions of the surplus: in this case Rubinstein-type of results break down (see van DAMME, SELTEN and WINTER [1990]). And second, the characterization of equilibria in the war of attrition turns out simpler if we use continuous time (first order conditions are necessary and sufficient).

3 The Model

Consider player A bargaining over one unit of surplus with her opponent B (A is player B's opponent!). An agreement can be reached at any time $t \in [0, \infty)$ but only three agreements, A, B or C are possible: agreement A gives player A a large share of the surplus, x^A, (while B gets only a small share $1 - x^A$); agreement B gives A a small share $x^B < x^A$ of the surplus (while B gets a large share $1 - x^B$); and agreement C is a compromise that gives A an intermediate share x^C, $x^B < x^C < x^A$, (while B gets also an intermediate share $1 - x^C$). If an agreement that gives x to A is reached at t, A gets a payoff $(x - a)\,e^{-t}$ and B gets a payoff $(1 - x - b)\,e^{-t}$ where a and b are the privately known realizations of two independent random variables. The ex-ante probability distribution of a and b are common knowledge and have a positive densities f and g on $[a_L,\, a_H]$ and $[b_L,\, b_H]$, with $a_H \leq x^A$ and $b_H \leq 1 - x^B$, that is, only bargainers that can get a positive surplus from some agreement are present.

The game starts with player A proposing agreement A, her opponent B proposing agreement B and bargaining goes on until proposals are compatible. If the proposals become more than compatible by the simultaneous concession of both players, then the exceeding surplus is evenly split. A *partial history at* t, h_t, is a full description of how the game has been played in the interval $[0,\, t)$. We say that $h_{t'}$ is a continuation of h_t if it coincides with h_t in $[0,\, t)$ and $t' > t$. We will write ϕ_t to denote a partial history in which no player has made a concession in $[0,\, t)$, and $A\,\tau_t\,(B\,\tau_t)$ to denote partial history in which player $A\,(B)$ proposed C at τ and kept it in $(\tau,\, t)$. H_t denotes the set of all partial histories at t.

A Δ-*reaction strategy for player* A is a measurable function

$$\varepsilon_A\ :\ [a_L,\, a_H] \times \cup_{t<\infty} H_t \to \{A,\, C,\, B\},$$

that is $\varepsilon_A\,(a,\, h_t)$ denotes A's proposal contingent on $(a,\, h_t)$. We require that ε_A satisfies:

(S.1) $\varepsilon_A\,(a,\, h_t) \in \{C,\, B\}$ if $\varepsilon_A\,(a,\, h_{t'}) = C$ if $h_{t'}$ is a continuation of h_t.

(S.1) $$\varepsilon_A\,(a,\, B\,t_{t+\delta}) = \varepsilon_A\,(a,\, \phi_t) \text{ if } \delta < \Delta.$$

Condition (S.1) requires that concessions are irreversible. This assumption is made for tractability and it implies that all h_t are either ϕ_t, $A\,\tau_t$, or $B\,\tau_t$. Condition (S.2) states that it takes some time to react to new concessions: Since $B\,t_{t+\delta}$ is the partial history at $t+\delta$ in which B proposes C at t, then $\varepsilon_A\,(a,\, B\,t_{t+\delta}) = \varepsilon_A\,(a,\, \phi_t)$ if $\delta < \Delta$ indicates that no concession by A can take place in $(t,\, t+\Delta)$ unless it had been planned prior to observing B's concession. We will think of the reaction lag Δ as negligible, and characterize the results in the limit as $\Delta \to 0$. However, it is necessary that $\Delta > 0$ so that the "first" date at which players react to proposal changes at t is well defined.

Let $\varepsilon = (\varepsilon_A,\, \varepsilon_B)$ denote a strategy profile and $u_a\,(\varepsilon,\, h_t)$ denote the (expected) utility that player A of type a obtains given the probability distribution over outcomes generated by $\varepsilon\,(h_t)$. [1]

A *system of beliefs* for A, π_A, maps each partial history into a probability measure on $[b_L,\, b_H]$ such that $\pi_A\,(h_0)$ has density g. A profile $\pi = (\pi_A,\, \pi_B)$ is consistent with ε if it is compatible with Bayes's Rule. Let $V_a(\varepsilon,\, h_t)$ denote the payoff that a expects from ε, conditional on $h_t\ :\ V_a\,(\varepsilon,\, h_t) = \int\, u_a\,(\varepsilon,\, h_t)\, d\pi_A\,(h_t)$.

A *Bayesian Equilibrium* (BE) is a strategy profile ε such that, $V_a\,(\varepsilon, \phi_0) \geq V_a\,(\varepsilon'_A, \varepsilon_B, \phi_0)$ and $V_b\,(\varepsilon, \phi_0) \geq V_b\,(\varepsilon_A, \varepsilon'_B, \phi_0)$ for all ε'_A and ε'_B, for almost all types. A *Perfect Bayesian Equilibrium* (PBE) is a strategy-belief profile $(\varepsilon,\, \pi)$ such that for all h_t, for almost all types $V_a\,(\varepsilon,\, h_t) \geq V_a\,(\varepsilon'_A,\, \varepsilon_B,\, h_t)$ and $V_b\,(\varepsilon,\, h_t) \geq V_b\,(\varepsilon'_A,\, \varepsilon'_B,\, h_t)$ for all ε'_A and ε'_B and π is consistent with ε.

Given a belief profile π and a partial history h_t, *the state of* h_t is the pair of proposals at h_t and the pair of beliefs $\pi_A\,(h_t)$, $\pi_B\,(h_t)$. A strategy-belief

1. Given a pair of types (a, b) and a history at t, each strategy profile ε generates a unique probability distribution over histories at t' for each $t' > t$, and hence a unique probability distribution over outcomes of the game (actually a unique history $h_{t'}$ and a unique outcome of the game if no simultaneous concessions are prescribed).

profile $(\varepsilon,\ \pi)$ is a *markov profile* iff, for all h_t, $\varepsilon_A(a,\ h_t)$ and $\varepsilon_B(b,\ h_t)$ depend only on a, b and the state of h_t. A *Markov Perfect Equilibrium* (MPE) is markov profile that is a PBE.

For any profile ε, play evolves so that, if player $A\,(B)$ proposes C, a new game of the same nature with only two agreements C and B (C and A) arises: The new game is a war of attrition that ends as soon as one of the two players makes a concession.

We refer to the game with only agreements A and B as the *war of attrition* and to the game with only agreements C and B (A and C) as the CB *war of attrition* (AC *war of attrition*).

4 Only Two Possible Agreements

Assume that C is unavailable. Player A needs to decide only how long to wait before she accepts agreement B and the game terminates as soon as one of the players concedes. Although a concession by A may reveal information on the value of a, this information is irrelevant because by the time B learns of A's concession the game is over. Thus if agreement C is impossible, we gain no insight into the subtle interactions between information and concessions that are inherent of the bargaining process. However, solving this simpler game is a necessary intermediate step before proceeding to the analysis of the model with three agreements for two reasons. First, because players may ignore the possibility of agreement at C, thus the BE strategies of the game in which C is unvailable are very relevant. Second, because if C is ever proposed by A before the end of the game, players face a continuation that is a CB war of attrition.

We can describe the war of attrition as in section 3, eliminating the possibility of a compromise agreement C. Proposition 1 fully characterizes the set of BE for two-agreement games [2]. Since the main objective of the present paper is to analyze games in which compromises are possible, and since a proof of Proposition 1 is long and involved, we skip it. (See PONSATI and SÁKOVICS [1995a] for a detailed proof).

PROPOSITION 1 : BE with only two agreements.
For games with only two agreements A and B, the set of BE outcomes is characterized by the following:

(i) If $a_H \leq x^B$ and $b_L < 1 - x^A < b_H$ then, almost surely, A concedes and agreement B is reached at 0.

2. Of course, what follows is equally valid for wars of attrition with agreements C and B or A and C.

(ii) If $a_L < x^B < a_H$ and $b_L < 1 - x^A < b_H$ the strategies are such that $\varepsilon^A(a, \phi_t) = B$ iff $a = \alpha(t)$ and $\varepsilon^B(b, \phi_t) = A$ iff $b = \beta(t)$, where α and β are increasing, differentiable functions uniquely solving the system of differential equations

$$\dot{\alpha} = \frac{(1 - F(\alpha))(1 - x^A - \beta)}{f(\alpha)(x^A - x^B)}, \qquad (1)$$
$$\dot{\beta} = \frac{(1 - G(\beta))(x^B - \alpha)}{g(\beta)(x^A - x^B)},$$

with $\lim_{t\to\infty}(\alpha(t), \beta(t)) = (x^B, 1 - x^A)$, and $(\lim_{t\to 0} \alpha(0) - a_L)$ $(\lim_{t\to 0} \beta(0) - b_L) = 0$.

(iii) If $a_H \leq x^B$ and $b_H \leq 1 - x^A$, then there is a continuum of BE characterized by the solution to (1), such that $(\lim_{t\to 0} \alpha(0) - a_L)$ $(\lim_{t\to 0} \beta(0) - b_L) = 0$.

Proof: see PONSATI and SÁKOVICS [1995a].

Remark: Proposition 1 fully characterizes the set of non trivial BE. If types are known to be incompatible, i.e. $a_L > x^B$ and $b_L > 1 - x^A$, then no player ever concedes. If it is known that player A cannot make a concession, i.e. $x^B < a_L$, while $b_L \leq 1 - x^A$, then B concedes at 0 if she is of type $b \leq 1 - x^A$.

Without getting into the details of the proof, we sketch the main argument. Note that types $a > x^B$ prefer perpetual conflict to B and therefore they would never yield in equilibrium. Thus, if A is believed to have a type $a > x^B$ with positive probability, she can credibly claim that she will never yield. So, if B is known to prefer A to perpetual disagreement (case (i)), A can take advantage of B and agreement A prevails. If both can credibly claim their unwillingness to yield, we get the classical war-of-attrition behavior, the players try to screen each other's type by prolonging the game and thus imposing a delay cost on their opponent (as well as themselves). Note that all types who derive positive utility even upon concession, eventually yield in equilibrium. If both players are known to prefer yielding to perpetual conflict we get a continuum of the wearing-down strategies mentioned above including the two limit cases in which one player yields at $t = 0$ with probability 1. The later are possible because now nothing can convince a player that her opponent will never concede. Therefore if A believes after any time without concession that B will soon concede, holding is a best response for A.

5 Games with Compromise

In this section we study games with three possible agreements, A, B, and C. The main result that we obtain are: a) that the outcome of the game in which A and B are the only possible agreements remains an equlbrium when the possibility of a compromise, C, is added; and b) that there exist equilibria with compromise that are (ex-ante) Pareto improving. The equilibrium profiles that allow compromise, however, cannot be markov profiles.

We consider first strategy profiles in which players ignore the possibility of reaching a compromise at C. A PBE with such profiles is characterized via a BE of the war of attrition. Given such a BE (possibly the unique one) let τ_a^w denote the time of concession by player A of type a, and recall that, for $t > 0$, $\beta(t)$ is the type of B that concedes at t. A *War of Attrition* (WA) *strategy-belief profile*, (ε^w, π^w), is defined as follows:

$$\varepsilon_A^w(a, h_t) = \begin{cases} A, & \text{for } h_t = \phi_t,\, t < \tau_a^w \text{ or } h_t = B\,\tau_t, \\ B, & \text{otherwise}, \end{cases}$$

$\pi_A^w(\phi_t)$ is the restriction of $\pi_A^w(\phi_0)$ to $[\beta(t), b_H]$, and $\pi_A^w(B\,\tau_t)$ assigns probability 1 to $b \leq \beta(t)$; ε_B^w and π_B^w are defined symmetrically.

That is, A keeps proposal A until the date at which she would concede B if C were unavailable. If A observes a concession to C by B, she keeps proposal A for ever. To support such behavior in a PBE it suffices that beliefs at partial histories $B\,\tau_t$, that occur with zero probability, assign probability one to types that can concede again [3].

PROPOSITION 2 : War of Attrition Equilibria (WAE).
A WA profile is a PBE.

Proof: By Proposition 1 (i), the continuation strategies after C is proposed are a BE. Since players of type $a > x^B$ cannot receive a positive payoff if they accept x^B, no deviation from the proposed strategy is profitable for them. Also, by Proposition 1, the proposed strategies prescribe equilibrium behavior for $a < x^B$, if they are playing the a game with two agreements A and B. Moreover, since B plays as if only A and B where possible, any strategy in which A proposes C at any t is strictly dominated by a strategy in which A directly proposes B. □

Proposition 3 that follows focuses attention on games in which the abilities to concede are very asymmetric: it is known that A gets a positive payoff even in agreement B while it is uncertain if B can afford agreement C. We show that in this case A immediately concedes B to B. This result is quite intuitive, only B can credibly claim that she needs a total concession, this ensures that A will eventually (at a finite t) concede to B. Given that the latest date at which A proposes B is finite, a backward induction argument proves that indeed it must be 0 [4].

3. This result can be generalized for games with n agreements that give A shares x^1 to x^n, $0 < x^1 < x^2 < \ldots < x^n < 1$, if $a_H > x^{n-1}$ and $b_H > 1 - x^{n-1}$.

4. This result can be generalized for games with n agreements, a generalization that is the analogue to the Coase conjecture in our context.

PROPOSITION 3 : Total Concession WAE.
If $a_H < x^B$ and $b_H > 1 - x^C$ then any PBE is a WAE in which A accepts B at $t = 0$.

Proof:

Step 1: A **never proposes** C**.** Note that if the game has not ended after she proposes C, by Proposition 1 (i), A must accept B immediately.Thus b will find profitable to propose C simultaneously with A. In this case, since A's proposal C will lead to proposal B after Δ, A is better off by proposing B directly.

Step 2: All a propose B **at some finite** t**.** Let $H_t(\tau)$ denote the probability that B concedes C or A by τ conditional on keeping proposal B in $[0, t)$, and let $\eta_t = \lim_{\tau \to \infty} H_t(\tau)$. If a never proposes B the she gets a payoff no greater than $\int_t^{\infty} (x^A - a)\, e^{-\tau}\, dH_t(\tau)$. Note that for each $t > 0$,

$$\int_t^{\infty} (x^A - a)\, e^{-\tau}\, dH_t(\tau) \leq (x^A - a)\, e^{-t}\, \eta_t.$$

Note also, that, since $b_H > 1 - x^C$, for t large enough η_t can be made arbitrarily small. Therefore, since $a < x^A$ there is some $t < \infty$ such that

$$\int_t^{\infty} (x^A - a)\, e^{-\tau}\, dH_t(\tau) \leq (x^A - a)\, e^{-t}\, \eta_t < (x^B - a)\, e^{-t},$$

that is, proposing B at t is better than not conceding at all.

Thus, $T = \mathrm{supremum}\ \{t$, such that a proposes B no later than t for $a \in [a_L, a_H]\} < \infty$.

Step 3: All a propose B **at 0.** Note that if $T > 0$, for small δ, no b will propose C or A in $(T - \delta, T]$, but then a proposing B at T will find profitable to deviate and propose it in $(T - \delta, T)$. Hence $T = 0$. □

Let us now explore the possibility that agreement C arises in equilibrium. We define a *Compromise Profile* (CP), as a pair (ε, π) that yields C with positive probability. A *Compromise Equilibrium* (CE), is a CP that is a PBE.

According with intuition, a player who gets higher benefits from agreement is willing to incur less delay cost in order to force a favorable outcome. Recall that this holds in the WAE because A's concessions to B are monotone in BE of the war of attrition, that is if $a < a'$ then $\tau_a^w < \tau_{a'}^w$. Although intuition suggests that concessions in CE should also be monotone, this cannot be guaranteed without an assumption on the behavior of indifferent types: we require that if a is indifferent between two proposals, then all weaker types must be proposing at most the least favorable one. This requirement, sated as Assumption 1, is sufficient to prove weak monotonicity of the concessions along any PBE. This fact, stated as Lemma 4, is crucial for characterizing the set of PBE because it implies that the support of beliefs π_A is an interval at all partial histories that occur along the play of a PBE.

Assumption 1

If a and $a'\,(a < a')$ are indifferent between proposing an agreement that assigns x and y to A, $x < y$, if a' proposes x, then a demands at most x.

LEMMA 4 : Monotonicity.

Let (ε, B) be a PBE that satisfies Assumption 1. For all h_t and for almost all $a < a'$ the following holds:
(i) $\varepsilon_A(a', h_t) = C \text{ implies } \varepsilon_A(a, h_t) \in \{B, C\}$,
(ii) $\varepsilon_A(a', h_t) = B \text{ implies } \varepsilon_A(a, h_t) = B$.

Proof: See Appendix.

Let us consider the implications of monotonicity if C is agreed with positive probability in equilibrium. First, it is necessary that weak types $(a < x^B)$ pool with some stronger types $(a > x^B)$; otherwise (by Proposition 1 (i)) a concession to C immediately forces them to a concession to B (and two immediately successive concessions cannot be an equilibrium behavior). Second, since different types of A separate as they propose C or not, there must be dates at which B, not seeing A propose C, must conclude that agreement at B is impossible. Since this information transmissions is instantaneous, B would like to react to it also instantaneously. However, if B were free to react instantaneously, inducing B's fast concession would be too cheap, thus upsetting the incentives of weak a's to propose C. Therefore, in order to ensure that separation via C occurs in an equilibrium, there must be a positive time interval without concessions that enforces the pooling and separation of types. We state this in the following Lemma 5.

LEMMA 5 : Partially pooling strategies are necessary in CE.

Consider a CE satisfying Assumption 1. Then (i) and (ii) hold for some ϕ_t, $0 \leq t < \infty$.
(i) Given $\underline{a} =$ infimum in the support of $\pi_B(\phi_t)$ and $\underline{b} =$infimum in the support of $\pi_A(\phi_t)$,

$$\varepsilon_A(a, \phi_t) = \begin{cases} C, \ a \in [\underline{a}, \bar{a}), \\ A, \ a \geq \bar{a}, \end{cases}$$

$$\varepsilon_B(b, \phi_t) = \begin{cases} C, \ b \in [\underline{b}, \bar{b}), \\ B, \ b \geq \bar{b}, \end{cases}$$

with $x^B \in [\underline{a}, \bar{a})$ and either $[\underline{b}, \bar{b}) = \varnothing$ or $x^B \in (\underline{b}, \bar{b})$.
(ii) There is $\bar{\mu} > 0$ such that:

$$\varepsilon_B(b, \phi_{t+\mu}) = B \text{ all } b \text{ and } \mu \in [0, \bar{\mu}),$$
$$\varepsilon_A(a, \phi_{t+\mu}) = A \text{ all } a \text{ and } \mu \in [0, \bar{\mu});$$

$$\varepsilon_B(b, \phi_{t+\bar{\mu}}) = C, \ b < 1 - x^C,$$

and, if $[\underline{b}, \bar{b}) \neq \varnothing$,

$$\varepsilon_A(a, \phi_{t+\bar{\mu}}) = C, \ a < x^C.$$

Proof: See Appendix.

A full characterization of MPE follows as a direct consequence of

Lemma 5. In fact, the set of MPE outcomes reduces to the set of WAE. If $\underline{a} > x^B$, and A does not offer a C at ϕ_t, then B knows that an agreement is possible only if she proposes at least C. The interval of time $(t+\Delta,\, t+\bar{\mu})$ must elapse before any new more by B, though (otherwise types $a < \underline{a}$, that must propose C at ϕ_t, would find profitable to deviate delaying their concession). Since no concessions take place during $(t+\Delta,\, t+\bar{\mu})$, players proposals are different at different dates while the state of the game remains unchanged, hence a CE is impossible with markov profiles.

PROPOSITION 6 : Markov Perfect Equilibria are WAE.
Assumption 1 implies that $(\varepsilon,\, \pi)$ is a MPE iff it is a WAE.

Proof: By Lemma 5 (ii) if $\mu \in [0,\, \bar{\mu})$, $\varepsilon_B(b,\, \phi_{t+\mu}) = B$ for all b, $\varepsilon_A(a,\, \phi_{t+\mu}) = A$ for all a and $\varepsilon_B(b,\, \phi_{t+\bar{\mu}}) = C$ for $b < 1 - x^C$. Note that $\varepsilon_A(a,\, \phi_{t+\mu}) = A$ for all a and $\mu \in [0,\, \bar{\mu})$ implies that the state is the same at $\phi_{t+\mu}$ and $\phi_{t+\bar{\mu}}$, yet $\varepsilon_B(b,\, \phi_{t+\mu}) \neq \varepsilon_B(b,\, \phi_{t+\bar{\mu}})$, contradicting that $(\varepsilon,\, \pi)$ is markov.

Since no PBE in markov profiles can yield agreement C, a MPE must be a WAE. □

We will now explore the existence CE in profiles that are not markov. For the sake of tractability we will restrict our analysis to games where some gains from agreement are possible and where a_H and b_H can only agree at A and B respectively: i.e. $a_L < x^B \leq 1 - b_H < x^C < a_H \leq x^A < 1 - b_L$.

Consider a strategy profile constrained as in Lemma 5, in which intervals of types $[\underline{a},\, \bar{a})$ and $[\underline{b},\, \bar{b})$ (with at least $[\underline{a},\, \bar{a}) \neq \emptyset$, simultaneously propose C. Note that the following are also necessary to support it as a PBE. First, at partial histories prior to ϕ_t players effectively play the war of attrition, thus a CE must prescribe dates of concession that are consistent with the unique (by Proposition 1 (ii)) BE of the war of attrition. Second, if only A proposes C at ϕ_t, the continuation $A\, t_{t+\Delta}$ is a CB war of attrition (if only B proposes C at ϕ_t, the continuation $B\, t_{t+\Delta}$ is a AC war of attrition) that has a unique (also by Proposition 1 (ii)) BE, ε must prescribe the same behavior as this BE.

The following Lemma 7 formally states these necessary conditions. Let t_a^{Aw} denote the time of concession of a in the CB war of attrition starting at $A\, t_{t+\Delta}$ with $a \in [\underline{a},\, \bar{a})$ and $b \in [\bar{b},\, b_H]$ (t_a^{Bw} is the time of concession of a in the AC war of attrition starting at $B\, t_{t+\Delta}$ with $a \in [\bar{a},\, a_H)$ and $b \in [\underline{b},\, \bar{b})$).

LEMMA 7 : Further necessary condition for CE.
Assume $a_L < x^B \leq 1 - b_H < x^C < a_H \leq x^A < 1 - b_L$. Consider a CE $(\varepsilon,\, \pi)$ satisfying Assumption 1 for which the conditions of Lemma 5 hold at ϕ_t. Then there must be $\bar{\rho}\,(\bar{\rho} > 0$ iff $t > 0)$ such that ε_A satisfies:

$$\varepsilon_A(a,\, \phi_\tau) = \begin{cases} A,\ \tau < t_a^w \\ B,\ \tau = t_a^w \end{cases} \qquad \text{for} \quad \tau \leq t - \bar{\rho},$$

$$\varepsilon_A(a,\,\phi_\tau) = A \quad \text{all } \ a \qquad \text{for} \quad \tau \in (t-\bar{\rho},\, t),$$

$$\varepsilon_A(a,\,B\,t_\tau) = \begin{cases} A, \ \tau < t_a^{Aw} \\ C, \ \tau = t_a^{Aw} \end{cases} \quad \varepsilon_A(a,\,A\,t_\tau) = \begin{cases} C, \ \tau < t_a^{Bw} \\ B, \ \tau = t_a^{Bw} \end{cases}$$
$$\text{for} \quad \tau \geq t+\Delta,$$

and ε_B satisfies the symmetric conditions.

Proof: Fix a CE satisfying Lemma 5 at ϕ_t.

Assume that $t > 0$. Note that any a that concedes earlier than t, by Lemma 4, reveals that $a < x^B$, thus she must be proposing B. Since a positive mass of types concedes C at t, there must be an interval of $(t-\bar{\rho},\, t)$ without concessions. Hence, play in $[0,\ t-\bar{\rho})$ would not change if C is removed from the set of feasible moves, thus concessions must take place as in the unique BE of the war of attrition.

It both players propose C at t the game is over. If only $A\,(B)$ proposes C at t, the game continues as a $CB\,(A\,C)$ war of attrition that satisfies the conditions of Proposition 1 (ii). Therefore, $\varepsilon_A(a,\ B\,t_\tau)$ and $\varepsilon_B(b,\ B\,t_\tau)$ ($\varepsilon_A(a,\ A\,t_\tau)$ and $\varepsilon_B(b,\ A\,t_\tau)$) are uniquely given by the BE strategies of the war of attrition.

Lemmas 5 and 7 give necessary conditions on the path of play along a CE. Moreover, since proposal C can take place only at ϕ_t or $\phi_{t+\bar{\mu}}$, it must be severely penalized at other dates: a sufficient penalty is that the opponent remains firm on her demand; such behavior is optimal if beliefs $\pi_B^w(h_t)$ assign probability 1 to $a < x^B$ for h_τ that do not occur along the play of ε. Interpreting t and $t+\bar{\mu}$ as pre-arranged or mediated meeting times provides some intuitive justification for such updating rules: it is not unreasonable that proposals of compromise occurring just before or just after the meeting dats are taken as proof of extreme weakness.

We no state sufficient conditions for a CP to be a CE.

PROPOSITION 8 : Compromise Equilibria.
Assume $a_L < x^B \leq 1 - b_H < x^C < a_H \leq x^A < 1 - b_L$.
Consider a profile $(\varepsilon,\ \pi)$ satisfying Assumption 1 and the necessary conditions of Lemma 5 and 7 for ϕ_t, $\underline{a} = \alpha(t)$, $\underline{b} = \beta(t)$, $\bar{a}$, $\bar{b}$, $\bar{\mu}$ and $\bar{\rho}$. Assume moreover that for $t' \neq t$ or $t' \neq t+\bar{\mu}$,

$$\varepsilon_A(a,\ B\,t'_\tau) = A \quad \text{for all} \ \ a, \ \text{and} \ \pi_A(b < x^B | B\,t'_\tau) = 1$$
$$(\text{and symmetrically for } B).$$

Then $(\varepsilon,\ \pi)$ is a CE iff
(8.a) $\bar{a}$ and $\bar{b}$ are such that the BE of the CB war or attrition starting at $A\,t_{t+\Delta}$ has a positive mass of b that concede C at $t+\Delta$ (if $\bar{b} > \underline{b}$, the AC war or attrition starting at $B\,t_{t+\Delta}$ has a positive mass of a that concede C at $t+\Delta$).

(8. b) $\bar{\mu}$ is such that $\bar{a}$ is indifferent between proposing C and A at ϕ_t (and similarly for $\bar{b}$ if $\bar{b} > \underline{b}$).
(8. c) If $t > 0$, $\bar{\rho}$ is such that $\underline{a} = \alpha(t)$ is indifferent between agreement B at $t - \bar{\rho}$ and proposing C at ϕ_t (and similarly for $\underline{b} = \beta(t)$ if $\bar{b} > \underline{b}$).

Proof: A deviation in which $A(B)$ proposes C at some $\tau \in [t,\, t + \bar{\mu})$ once she believes that $a > x^B\,(b > 1 - x^A$ is not profitable, since $B(A)$ remains firm for ever if C is offered at $t' \neq t$ or $t' \neq t + \bar{\mu}$. The later behavior is otpimal given that $B(A)$ believes that a $a < x^B\,(b < 1 - x^A)$, with probability 1.

Assume that $\phi_t = \phi_0$, $\underline{a} = a_L$ and $\underline{b} = b_L$. We will prove that (8.a) and (8.b) are necessary and sufficient for the $(\varepsilon,\, \pi)$ profile to be a CE. (With condition (8.c), the generalization to ϕ_t with $t > 0$, $\underline{a} = \alpha(t)$ and $\underline{b} = \beta(t)$ is immediate.)

Assume (8.a) fails and the probability that B concedes C at Δ in the CB war of attrition starting at $A\,0_\Delta$ is zero. For a is close to a_L, a's expected payoff from the proposed strategy is arbitrarily close to $(x^C - a)\,G(\bar{b}) + (1 - G(\bar{b}))(x^B - a)\,e^{-\Delta}$. Consider the following class of deviations: do not concede at 0, concede C at μ, and B at $2\,\mu$, $0 < \mu < \Delta$. As $\mu \to 0$, this strategy gives a payoff arbitrarily close to $(x^C - a)\,G(\bar{b}) + (1 - G(\bar{b}))(x^B - a)$. Hence, for each $\Delta > 0$, there is some non-zero measure subset of types who would deviate.

Observe that for $t = 0$, (8.b) is equivalent to:

$$\begin{aligned} G(\bar{b}) + (1 - G(\bar{b})) \int_{\bar{b}}^{1-x^c} e^{-\tau_b^{AW}}\, d\pi^A(A\,0_\Delta) \\ = G(\bar{b})\,e^{-\Delta} + (G(1 - x^C) - G(\bar{b}))\,e^{-\bar{\mu}}. \end{aligned}$$

Since $\bar{a} > x^B$ does not concede in the CB war of attrition starting at $A\,0_\Delta$, the expected value of this continuation for all $a > x^B$ is $(x^C - a)\int_{\bar{b}}^{1-x^c} e^{-\tau_b^{AW}}\, d\pi^A(A\,0_\Delta)$. If $\bar{a}$ proposes A at 0 and B proposes C, $\bar{a}$ must respond with C at Δ. If $\bar{a}$ proposes A at 0 and B proposes B, she must wait until $\bar{\mu}$ and then offer C. That is, $\bar{a}$'s indifference requires that

$$\begin{aligned} (x^C - \bar{a})\,[G(\bar{b}) + (1 - G(\bar{b})) \int_{\bar{b}}^{1-x^c} e^{-\tau_b^{AW}}\, d\pi^A(A\,0_\Delta)] \\ = (x^C - \bar{a})\,[G(\bar{b})\,e^{-\Delta} + (G(1 - x^C) - G(\bar{b})\,e^{-\bar{\mu}}]. \end{aligned}$$

Moreover this implies that all $a \in (x^B, x^C)$ are indifferent between proposals C or A at 0. Hence if (8.b) fails, either all types $s \in (x^B,\, x^C)$ would want to propose x^B at 0, or none would, and neither of these two siuations can prevail in a CE.

Next we prove that if (8.a) and (8.b) hold, then no player can find profitable to deviate. If only A proposes C at 0, then there must be some $\tilde{b}$ such that all b, $\bar{b} \leq b < \tilde{b}$, accept C at Δ. Thus $a > 0$, receives at least $G(\bar{b})(x^C - a) + (G(\tilde{b}) - G(\bar{b}))(x^C - a)\,e^{-\Delta} + (1 - G(\tilde{b}))\,(x^B - a)\,e^{-\Delta} > G(\bar{b}))(x^C - a) + (1 - G(\bar{b}))(x^B - a)$. That is, here is no $\mu > 0$ such that condeding C at μ and B at 2 μ is a profitable deviation. Let us check that there is no other profitable deviation:

(i) Types $a > x^C$, are playing a dominant strategy.

(ii) Let $a < x^B$ and assume she does not concede in $[0, \Delta)$. If B proposes C at 0, the best any $a < x^B$ can do is to accept it, which yields a payoff $(x^C - a)\, e^{-\Delta}$. If B does not proposes C at 0, then the best a can do is either to propose x^B at Δ, or to wait until $\bar{\mu}$ to propose C. The payoff under the first deviation is $G(\bar{b}))(x^C - a)\, e^{-\Delta} + (1 - G(\bar{b}))(x^B - a)\, e^{-\Delta}$. The payoff under the second is $G(\bar{b}))(x^C - a)\, e^{-\Delta} + (G(x^C) - G(\bar{b}))(x^C - a)\, e^{-\bar{\mu}}$. Both are lower than the payoff that receives along ε.

(iii) Consider $x^B < a < x^C$. By (8.b), a is indifferent between offering C at 0 or waiting to Δ or $\bar{\mu}$. Clearly, offering C later than $\bar{\mu}$ can never increase their payoff. If offering C at t, $0 < t < \bar{\mu}$, never receives acceptance. Therefore this deviation is not profitable: a cannot expect more than $G(\bar{b}))(x^C - a)$ which is strictly less than the payoff she obtains along ε. □

Having proved that CE exist, we point out that if agreements A and B are very extreme, then the CE (ex-ante) Pareto Superior to the WAE.

PROPOSITION 9 : CE Pareto Dominate WAE.
As $x^B\,(1 - x^A) \to 0$, the CE are (ex-ante) Pareto Superior to the WAE.

Proof: The expected gains of A from the WAE are bounded above by $G(1 - x^A)$ for all a. The expected gains from a CE are bounded below by zero for $a \geq x^C$, and by $G(1 - x^C)(x^C - a)\, e^{-(t+\bar{\mu})}$ for $a < x^C \cdot G(1 - x^A) \to 0$ as $x^A \to 1$, while $G(1 - x^C)\, E\{x^C - a | a < x^C\}\, e^{-(t+\bar{\mu})}$ remains bounded above zero. □

We conclude this section by computing the case of symmetric agreements $x = x^B = 1 - x^A$ and $x^C = 1/2$ and uniformly distributed types on $[0, 1 - x]$. In this case, symmetric CE with compromises at $t = 0$ exists if and only if $x^B = 1 - x^A < 1/4$. That is, when the war of attrition equilibrium is most inefficient because of a very small probability of agreement, then there are symmetric, semi-pooling PBE in which compromises are reached at 0 with positive probability. The intuition is simple: Consider the CB war of attrition after A has proposed 1/2 at $t = 0$. If x^B is close to 1/2, the probability that A can concede again to B is much higher than the probability that B can accept 1/2, thus we cannot expect that a positive mass of b's will respond to A's concession to C with another concession to C.

Example : Uniformly Distributed Types.

(I) BE of the war of attrition with uniform distributions:

Consider the BE in a war of attrition with agreements that assign to A x and y, $x < y$, and where a and b are uniformly distributed in $[a_L, a_H]$ and $[b_L, b_H]$, $a_L < x < a_H$, $b_L < 1 - y < b_H$.

A straightforward computation shows that for all $t > 0$, $\beta(t) = K + \alpha(t)$. With $\lim_{t \to \infty}(\alpha(t), \beta(t)) = (x, 1 - y)$, $(\alpha(t), \beta(t))$ must be such that:

(i) For all $t > 0$, $\frac{a_H - x}{a_H - \alpha(t)} = \frac{b_H - (1-y)}{b_H - \beta(t)}$, that is, for all $t > 0$, the probability that $a > x$ must be equal to the probability that player $b > 1 - y$.

(ii) A concedes at $t = 0$ with positive probability iff $\frac{a_H - x}{a_H - a_L} < \frac{b_H - (1-y)}{b_H - b_L}$.

(II) Symmetric CE with $t = 0$ when $x = x^B = 1 - x^A$, $x^C = 1/2$, a and b uniform on $[0,\, 1 - x]$:

A symmetric CE with $t = 0$ exists if there are $\bar{a} = \bar{b}$, $x < \bar{a} < 1/2$, and $\bar{\mu} > 0$, such that (8.a) and (8.b) hold. For each $\bar{a}$, $x < \bar{a} < 1/2$, there is some $\bar{\mu} > 0$ which satisfies (8.b). Hence it suffices to see that there is some $\bar{a}$ such that (8.a) holds. By part (I) a positive measure of b concedes y at 0 iff $\frac{a_H - x}{a_H - a_L} < \frac{b_H - (1-y)}{b_H - b_L}$. Substituting $y = 1/2$, we find that B concedes with positive probability at $A\,0_\Delta$ iff $\frac{\bar{a} - x}{\bar{a}} < \frac{1/2 - x}{1 - x - \bar{b}}$. This inequality is satisfied for some $\bar{a} = \bar{b} \in (x,\ 1/2)$ iff $x < 1/4$.

6 Conclusion

A model of the bargaining process as a two-stage concession game in continuous time with two-sided private information has been presented. It has been proved elsewhere that, in games in which no compromises are possible, strict uncertainty on the ability of players to make concessions implies the existence of a unique equilibrium. Here we show that if an intermediate option is added to the two extreme agreements, the outcome of the original unique equilibrium persists as an equilibrium outcome of the more complex game. However, the uniqueness of the equilibrium does not necessarily persist.

For games with only three agreements and types that have full support, the whole set of PBE has been characterized. In addition to the equilibria that reproduce the war of attrition, WAE, we have provided necessary and sufficient conditions for existence of Compromise Equilibria (CE), these are semi-pooling equilibria in which both parties make concessions with positive probability and compromises are reached after perhaps some delay.

The characterization of these equilibria rely heavily on the particularly simple structure of the equilibrium strategies in the continuation after a concession takes place, that is, on the structure of the set of BE in games with only two agreements: Under the assumption that all types that can extract some surplus are present in the negotiation, any two agreement game that arises as a continuation subgame has a unique BE. Such an assumption, however, is of no help in games with more than three agreements because PBE outcomes with three agreements are far from being unique.

The driving force behind our results is the fact that players revealing ability to make concessions are forced to make them. As a consequence, players refrain from partial concessions for fear of being identified as weak. Thus, even in the limited framework of games with at most three agreements, we provide a qualitative description of bargaining behavior with two-sided incomplete information that is robust. First, the persistence of equilibria in which players follow strategies that ignore the possibility of compromise can be generalized to games with many possible agreements. Second, the structure of the equilibria in which compromises take place, highlights the strong incentive constraints imposed by the process of information transmission in sequential mechanisms: in order to get equilibria in which intermediate agreements take place it is necessary that weak types be separated from strong types, but enforcing such separation requires positive intervals of time without concessions. Finally, our results suggest that third parties enhance the efficiency of bargaining outcomes by restricting the opportunities for too much direct communication and coordinating the timing of concessions.

Thus, our work leaves open two different lines for further research: obtaining new results for games with n possible agreements and studying the effects of explicitly introducing a mediator in the bargaining process, PONSATI and SÁKOVICS [1995b] explores this direction.

LEMMA 4 : Monotonicity
Let $(\varepsilon,\, \pi)$ be a PBE that satisfies Assumption 1. For all h_t and for almost all $a < a'$ the following holds:
(i) $\varepsilon_A\,(a',\, h_t) = C$ implies $\varepsilon_A\,(a,\, h_t) \in \{B,\, C\}$,
(ii) $\varepsilon_A\,(a',\, h_t) = B$ implies $\varepsilon_A\,(a,\, h_t) = B$.

Proof: Fix a PBE $(\varepsilon,\, \pi)$. Let P and P' denote two agreements that give A shares x and x', $x > x'$. Consider h_t, at which A's proposal is P. Let $V_a\,(h_t,\, P')$ denote a's expected payof at h_t if she decides to switch to proposal P' at t, and let $V_a\,(h_t,\, P)$ denote a's expected payof at h_t if she decides to keep proposal P at t; observe that $V_a\,(h_t,\, P')$ and $V_a\,(h_t,\, P)$ are continuous and strictly decreasing in a. Also, let $\eta\,(a,\, h_t,\, P)$ denote the a's expected value of the discounting factor $e^{-\tau}$, at h_t conditional on a keeping proposal P at h_t

Claim 1 : If $V_a\,(h_t,\, P) \leq V_a\,(h_t,\, P')$ for some a then $V_{a'}\,(h_t,\, P) \leq V_{a'}\,(h_t,\, P')$ for all $a' < a$. It is easy to see that there are always types who would get a negative payoff from a proposal P' and a non-negative payoff from proposal P. Therefore, by continuity, $\exists\, a^*$ such that $V_{a^*}\,(h_t,\, P) = V_{a^*}\,(h_t,\, P')$. Take $a' < a^*$. We know that since $V_a\,(h_t,\, P)$ is strictly decreasing in a, $V_{a'}\,(h_t,\, P) > V_{a^*}\,(h_t,\, P')$. Moreover, note that $V_{a'}\,(h_t, P) - (a^* - a')\,\eta\,(a', h_t, P) \cdot V_{a^*}\,(h_t, P)$. This is so, because the LHS is what a^* could get imitating a'. Now, $V_{a^*}\,(h_t,\, P) + (a^* - a')\,\eta\,(h_t,\, P) = V_{a^*}\,(h_t,\, P') + (a^* - a')\,\eta\,(a',\, h_t,\, P) < V_{a'}((h_t,\, P) + (a^* - a')\,\eta\,(a',\, h_t,\, P)$ and therefore $V_{a'}\,(h_t,\, P') > V_{a'}\,(h_t,\, P) - (a^* - a')\,\eta\,(a',\, h_t,\, P)$. Hence, for a' close to a^*, $V_{a'}\,(h_t,\, P') \geq V_{a'}\,(h_t,\, P)$. If the inequality is strict for all $a' < a^*$, the claim is proved. If not, the argument can be repeated starting at a' for which $V_{a'}\,(h_t,\, P') = V_{a'}\,(h_t,\, P)$, etc.

A similar argument proves:

Claim 2 : If $V_a\,(h_t,\, P) = V_a\,(h_t,\, P')$ for some s then $V_{a'}\,(h_t,\, P) \geq V_{a'}\,(h_t,\, P')$ for all $a' > a$.

By claims 1 and 2, if beliefs before t had support $[a_t,\, a_H]$ then the following holds true:

$$V_a\,(h_t,\, P) < V_a\,(h_t,\, P') \qquad \text{if} \quad a \in [a_t,\, \underline{\omega}),$$

$$V_a\,(h_t,\, P) = V_a\,(h_t,\, P') \qquad \text{if} \quad a \in [\underline{\omega},\, \bar{\omega}],$$

$$V_a\,(h_t,\, P) > V_a\,(h_t,\, P') \qquad \text{if} \quad a \in (\bar{\omega},\, a_H],$$

where $a_t \leq \underline{\omega} \leq \ \bar{\omega} < a_H$.

Given Assumption 1, we get the desired result.

LEMMA 5 : Partially pooling strategies are necessary in CE.

Consider a CE satisfying Assumption 1. Then (i) and (ii) hold for some $\phi_t,\ 0 \leq t < \infty$.

(i) Given $\underline{a} =$ infimum in the support of $\pi_B(\phi_t)$ and $\underline{b} =$ infimum in the support of $\pi_B(\phi_t)$,

$$\varepsilon_A(a,\ \phi_t) = \begin{cases} C,\ a \in [\underline{a},\ \bar{a}), \\ A,\ a \geq \bar{a}, \end{cases}$$

$$\varepsilon_B(b,\ \phi_t) = \begin{cases} C,\ b \in [\underline{b},\ \bar{b}), \\ B,\ b \geq \bar{b}, \end{cases}$$

with $x^B \in (\underline{a},\ \bar{a})$ and either $[\underline{b},\ \bar{b}) = \varnothing$ or $x^B \in (\underline{b},\ \bar{b})$.

(ii) There is $\bar{\mu} > 0$ such that:

$$\varepsilon_B(b,\ \phi_{t+\mu}) = B \text{ all } b \text{ and } \mu \in [0,\ \bar{\mu}),$$
$$\varepsilon_A(a,\ \phi_{t+\mu}) = A \text{ all } a \text{ and } \mu \in [0,\ \bar{\mu});$$

$$\varepsilon_B(b,\ \phi_{t+\bar{\mu}}) = C,\ b < 1 - x^C,$$

and, if $[\underline{b},\ \bar{b}) \neq \varnothing$,

$$\varepsilon_A(a,\ \phi_{t+\bar{\mu}}) = C,\ a < x^C.$$

Proof: By Lemma 4, any strategy in which the time of the first concession is fully revealing must be strictly monotone. By Proposition 1, in fully revelaing strategies, any concession by $a < x^B$ leads to a total concession.

Therefore if C arises it must be that for some ϕ_t there is a set of a's, A_t, of positive measure, that propose C.

By Lemma 4, for each h_t, beliefs must have support on an interval. In particular, as long as no concession is observed, beliefs have support on some interval $(a_t,\ i_H]$, with $a_L \leq a_t$, and $a_{t'} \leq a_t$ for all $t' < t$. Also by Lemma 4, A_t must be an interval. Let $\bar{a}_t$ denote the supremum of A_t. Consider the following cases:

Case I : $\bar{a}_t \leq \underline{x}^B \cdot B$ best response must be to keep proposal B with probability 1, a concession leads to a CB war of attrition in which A must concede B immediately. This cannot occur in a PBE since any $a \in A_t$ would be better off proposing B directly.

Case II : $\bar{a}_{\underline{t}} > \underline{x}^B$ *and* B *remains firm with probability* 1. If A does not concede C she gives a final proof that she is $a > x^B$, thus B must at least concede C in response. B's response, however, must not be at $t + \Delta$. If B's response is not delayed beyond $t + \Delta$, a deviation is profitable for any $a \in A_t$: not conceding C at ϕ_t pays at least $\pi^A(1-b > x^C|\phi_t)(x^C - s)e^{-\Delta}$, while the payoff if the concedes is strictly below $\pi^A(1-b > x^C|\phi_t)(x^C - a)$ (because she gets the expected payoff of the BC war of attrition in which not all $1 - b > x^C$ propose C immediately), as $\Delta \to 0$ the former expression must be greater than the later.

Case III : $\bar{a}_{\underline{t}} > \underline{x}^B$ *and* $b \in B_{\underline{t}} \neq \emptyset$, *proposes* C *at* $\phi_{\underline{t}}$, *with* $\bar{b}_{\underline{t}} = \sup\ B_{\underline{t}} > 1 - x^A$. There is agreement C *on* x^2 at t, or the continuation is one of the following: $\phi_{t'}\, t' > t$ and beliefs that C is the unique possible agreement, an AC war of attribution or a CB war of attrition. In any case, if A does not offer C, B beliefs with probability 1 that future agreements are possible only if she offers C. Yet B must not offer C at $t + \Delta$; otherwise any $a \in A_t$ has a profitable deviation: not conceding C at ϕ_t pays at least $\pi^A\,(1 - b > x^C|\phi_t)\,(x^C - s)\,e^{-\Delta}$ (with $\Delta \to 0$), while the payoff is she concedes is strictly less than $\pi^A\,(1 - b > x^C|\phi_t)\,(x^C - a)$ (because proposal C is followed, with positive probability, by a CB war of attrition).

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