Economic Theory **11**, 667–672 (1998)

Economic Theory

Rubinstein bargaining with two-sided outside options★

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Received: February 27, 1996; revised version: March 28, 1997

Summary. In this note we show that if in the standard Rubinstein model both players are allowed to leave the negotiation after a rejection, in which case they obtain a payoff of zero, then there exist a continuum of subgame-perfect equilibrium outcomes, including some which involve significant delay. We also fully characterize the case in which, upon quitting, the players can take an outside option of positive value.

JEL Classification Number: C78.

1 Introduction

Ariel Rubinstein's infinite horizon, alternating offers bargaining game (Rubinstein, 1982) has been consecrated as the fundamental extensive form for non-cooperative bargaining games. The main feature for which it has reached such a level of prominence resides in the fact that it provides a unique solution (subgame-perfect equilibrium), which moreover is efficient.

The uniqueness of equilibrium property has been shown to be impervious to the relaxation of the timing of offers (as long as simultaneous offers are not permitted, see Sákovics, 1993). However, under a number of other circumstances, this property no longer holds: if the set of possible partitions is made discrete rather than continuous (smallest money unit, see van Damme, Selten and Winter, 1990), if preferences are non-stationary (non-exponential discounting, see Binmore, 1987) or if players have an enlarged set of actions (strikes, see Haller and Holden, 1990, or "money burning" in general, see

★ C. Ponsatí acknowledges support from DGICYT through project PB-92-0590 and CIRIT through project GRQ-93-2044. J. Sákovics acknowledges support from the HCM program of the EC through contract CHRX-CT94-0489 "Games and Markets" and from DGICYT through project PB-93-0679. We are grateful to Margarida Corominas for pointing out an error in a previous version of this paper.

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Avery and Zemsky, 1994) available to them. Whilst the empirical relevance of each of these modifications can be argued, in this note we highlight a more fundamental reason why there could be a large set of subgame-perfect equilibria in the alternating offers bargaining game. We show that if the bargainers are not "locked in," as Rubinstein assumed, but they can freely quit whenever they wish so, then even if their outside option is zero (that is, they do not have one), there exist a continuum of subgame-perfect equilibria.[1]

The relevance of outside opportunities for the outcome of a negotiation may sound obvious, but it was not before the mid-eighties that the intrinsic difference between a "threat-point" and an outside option (the Outside Option Principle, see Shaked and Sutton, 1984) was clearly established. Perhaps by chance, all the initial work on incorporating outside options into the Rubinstein game assumed that only the responder had the opportunity to take up her option. In this case, if the responder's option is larger than her equilibrium share in the game without outside options, there is a unique subgame-perfect equilibrium, in which the player who has the option obtains a payoff equal to the value of that option. Otherwise, the option has no effect on the outcome. It was Shaked (1987/1994) who recognized that the assumption that only the responder can opt out is not without loss of generality. In fact, if it is the proposer who can threaten to take his outside option, the strategic consequences are markedly different. This is so because, as long as his threat is credible (that is, his outside option exceeds his continuation value if he does not leave the game), he can appropriate the entire surplus by making a take-it-or-leave-it offer. As Shaked shows (see also Osborne and Rubinstein, 1990), when one of the players may take up his option in the periods where he is the proposer, there exist a range of outside options (strictly between zero and one) for which there exist multiple equilibria.

What we show in this note is that something has remained overlooked. This is that the restriction that only one of the players has the opportunity to take up her option also entails a significant loss of generality.

2 The model

Player 1 and Player 2 are bargaining over the division of a fixed surplus, normalized to one. Player i ($i = 1, 2$) has an outside option, $0 \leq x_i$, available in case they break up negotiations. Time runs in discrete periods of equal length, numbered by the natural numbers. In even (odd) periods Player 1 (Player 2) makes an offer. The other party may accept, thus terminating the game with agreement at the proposed shares. Alternatively, if he rejects, either of the two parties may take their outside option, in which case both of them receive their outside payoff. If the offer is rejected but neither player

[1] While our result confirms the intuition of Avery and Zemsky (1994), it does not follow either from their general characterization (which does not allow for outside options) or from their specific treatment of outside options, which builds on Shaked's (1987/1994) model.

opts out then bargaining goes on to the following round. Player i ($i = 1, 2$) discounts future payoffs via the discount factor δ_i. Consequently, if they reach the agreement (y, $1 - y$) in period t, Player 1's payoff is $y\delta_1^t$, while Player 2's payoff is $(1 - y)\delta_2^t$. Similarly, if either of the players decides to take her outside option in period r then their payoffs are $(x_1\delta_1^r, x_2\delta_2^r)$.

3 The solution

Let us start the analysis of the model by noting that, if the sum of the two options is greater than the surplus to be divided, the unique Nash equilibrium outcome involves both players taking their outside option in period zero, since no agreement can be Pareto optimal (recall that it is sufficient that a single player opt out for obtaining this outcome) and if they take the options later their payoff is discounted. Basically in such a case the negotiation does not even begin, since in fact there do not exist "gains from trade." Therefore, for the rest of this note we assume that $x_1 + x_2 \leq 1$.

Our main result is a consequence of the following simple lemma:

Lemma 1 *For any* $0 \leq x_1 \leq 1 - x_2 \leq 1$, *immediate agreement at* $(1 - x_2, x_2)$ *is an outcome that can be supported by a subgame-perfect equilibrium.*

Proof. Consider the following strategies: if Player i is the proposer he always asks for $1 - x_i$; the responder accepts any proposal that is not worse than the (candidate) equilibrium proposal; if the proposer asks for more, then the responder rejects and takes her outside option; if the responder does not accept a proposal the proposer opts out. It is straightforward to verify that these strategies constitute a subgame-perfect equilibrium. Q.E.D.

The intuition for the difference between this model and the case in which only one player can opt out is easy to understand. The fact that the proposer tomorrow has large bargaining power commits her to exploit it in equilibrium. Therefore, today's proposer must expect little tomorrow and thus his threat to take his option today is credible. With one-sided options, these are not always credible, since the owner of the option can obtain a decent payoff even when he is responder. This is so, because next period's proposer does not have an outside option and therefore the responder has some bargaining power, since she controls whether payoffs are to be discounted. When both players can opt out when they are proposers, the proposers can commit to opting out, thus depriving the responders of all their bargaining power – except for the one given to them by the outside option principle – in turn making the commitment to opt out credible. Thus, the result in Lemma 1 depends exclusively on the possibility for both players to opt out when they are proposers, the responders' opting out only serves as "damage control," guaranteeing a minimum payoff to the second offeror.[2]

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[2] This explains our laxity, when stating the extensive form of our model: whether the players' opting out decision upon a rejection is taken simultaneously or sequentially does not matter.

Let us state this formally. Assume that each player can opt out only in the periods when she is the proposer and her offer has been rejected. Then the following is true:

Lemma 2 *Even if only the proposer can opt out in every period, for any* $0 \leq x_1 \leq 1 - x_2 \leq 1$, *immediate agreement at* $(1 - x_2, x_2)$ *is an outcome that can be supported by a subgame-perfect equilibrium.*

Proof. Consider the following strategies: the proposer always asks for 1; the responder accepts any proposal; if the responder rejects a proposal the proposer opts out. It is straightforward to verify that these strategies constitute a subgame-perfect equilibrium. Q.E.D.

If the options of the players are high then they must take them in equilibrium and thus the equilibrium characterized in the corresponding lemma is unique. Otherwise, we have multiple equilibria, which are sustained by a threat to switch to the extreme equilibria of the Lemmas. In the following theorem we give a complete characterization of the equilibria of our original – and perhaps more realistic – model. A qualitatively similar result holds true for the case when only the proposers can opt out.

Theorem *i) If* $x_i \leq \delta_i^2(1 - x_j/\delta_j)$ *the outcomes that can be supported by a subgame perfect equilibrium are either immediate efficient agreements that give Player 1 a payoff in* $[1 - \delta_2(1 - x_1/\delta_1), 1 - x_2]$ *or, for any period* $t > 0$, *any efficient agreement that gives Player 1 a share in* $[(1 - \delta_2(1 - x_1/\delta_1))\delta_1^{-t}, 1 - (1 - \delta_1(1 - x_2/\delta_2))\delta_2^{(1-t)}]$, *whenever this interval is non-empty.*

ii) Otherwise the equilibrium outcome characterized by Lemma 1 is unique.

Proof. Note that the only way to end the game in the first period at an outcome different from the one characterized by Lemma 1 is for Player 2 to accept Player 1's initial proposal. For the lowest possible share for player 1 in such an equilibrium we need three conditions to hold: a) Player 2 should be indifferent between accepting and waiting for next period, since otherwise either she could obtain more in equilibrium by threatening to wait, or it would not be credible for her to threaten to reject a slightly smaller offer; b) both players should (weakly) prefer this agreement to their option; and c) Player 1 should weakly prefer continuing to opting out, since otherwise Player 2's threat to reject in case Player 1 deviates would not be credible. To derive explicitly the necessary conditions for the existence of such an equilibrium, observe that, by c), the continuation value of Player 1 from the next period must be at least x_1/δ_1. This, in turn, implies that, by a), Player 2's maximum possible share is $\delta_2(1 - x_1/\delta_1)$. By symmetry, this share puts an upper bound on both continuation values, that is, we need $x_1/\delta_1 \leq \delta_1(1 - x_2/\delta_2)$ and $x_2/\delta_2 \leq \delta_2(1 - x_1/\delta_1)$. Given these conditions (b) is always satisfied. It is straightforward to see that these conditions are also sufficient: Table 1 describes[3] the equilibrium strategy

[3] See Osborne and Rubinstein (1990) for a detailed explanation of how to describe strategies as automata.

Table 1. The equilibrium strategies.

	Base state	Threat
1 proposes	$(z, 1-z)$	$\left(1-\frac{x_2}{\delta_2}, \frac{x_2}{\delta_2}\right)$
1 accepts	–	$y \geq \frac{x_1}{\delta_1}$
2 proposes	–	$\left(\frac{x_1}{\delta_1}, 1-\frac{x_1}{\delta_1}\right)$
2 accepts	$y \geq 1-z$	$y \geq \frac{x_2}{\delta_2}$
1 opts out	iff he hasn't deviated	iff he is proposer
2 opts out	No	Iff she is proposer
Transitions	Go to Threat if 1 deviates	Absorbing

profiles supporting immediate agreement, giving Player 1 a share of $z \in [1-\delta_2(1-x_1/\delta_1), 1-x_2]$; y denotes a generic share offered to the responder.

The delayed agreements are supported by strategies where up to the equilibrium date of agreement only non-serious[4] offers are made. Note that the most effective deviation by any player is the one made in the first period she can propose. The most stringent retaliation for this deviation is if in the next period they switch to the extreme equilibrium. As we have seen above, this threat is only effective if the deviant offer is at least $1-\delta_2(1-x_1/\delta_1)$. Consequently, any offer by Player 1 in the first period which is less than $1-\delta_2(1-x_1/\delta_1)$ is a valid threat, and therefore any equilibrium outcome has to give at least that much to him. Hence the lower bound on his equilibrium payoffs. To calculate the upper limit, the same argument can be used for Player 2 in the second period. Q.E.D.

The following corollary is now immediate:

Corollary *If in the standard alternating offers game the players are allowed to quit at any time, in which case their payoffs are zero, then the outcomes that can be supported by a subgame-perfect equilibrium are either immediate efficient agreements that give Player 1 a payoff in* $[1-\delta_2, 1]$ *or, for any period* $t > 0$, *any efficient agreement that gives Player 1 a share in* $[(1-\delta_2)\delta_1^{-t}, 1-(1-\delta_1)\delta_2^{(1-t)}]$, *whenever this interval is non-empty.*

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[4] For example, the proposer always asks for one and the responder rejects anything that gives her less than one.

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