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# Stakeholder bargaining games

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**Abstract** We study bilateral bargaining problems with an interested third party, the stakeholder, that enjoys benefits upon a bilateral agreement. To address the strategic implications of stakeholders over negotiations, we consider a model where two bargainers interact in the presence of a third party that (a) can transfer a share of her benefits to the bargainers but cannot receive a share of the bilateral surplus, and (b) while she may not participate in all periods of the negotiation, she cannot remain entirely inhibited. Our main findings are: (1) the stakeholder's (reverse) liquidity constraint implies the existence of a multiplicity of stationary subgame perfect equilibria that include outcomes with very asymmetric bilateral agreements, and (2) the partial participation of the stakeholder may be the source of severe inefficiency.

**Keywords** Bargaining · Stakeholders · Public sector labour · relations · Third party intervention

## 1 Introduction

Bargaining between two parties often affects *stakeholders*, third parties who care for the resolution of the conflict, and yet are unable to impose an agreement upon the contending bargainers. We model such bargaining problems as non-cooperative games with three players: two bargainers, who must reach an agreement on how to split one unit of bilateral surplus, and a third player, the stakeholder, who enjoys a

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positive externality upon the bilateral agreement. Two specific features distinguish stakeholder bargaining games from regular trilateral negotiations. The first is the (reverse) liquidity constraint that affects the stakeholder: she can transfer a share of her benefits to the bargainers but cannot receive a share of the bilateral surplus. The second is that while the stakeholder cannot commit to remain inhibited, her participation is not assured in every period of the negotiation either.

Scenarios of this type abound in industrial relations. In sectors of public interest the disruption of essential services like public transport, air traffic control, hospitals or the supply of utilities has a substantial impact on the population at large. Correspondingly, the government's stake in bilateral conflicts which are of public concern is of great consequence. In these situations, while the government might be willing to provide a handout to foster agreement, claiming a share of the bilateral surplus under negotiation is not an option. Our model also applies to international relations. For example, in an armed conflict that threatens the political stability of a region extending beyond the geographically disputed borders, the international community is a stakeholder. Third party intervention often takes the form of positive transfers (aid) to facilitate agreements; transfers from the contender to the international organizations are out of the question.

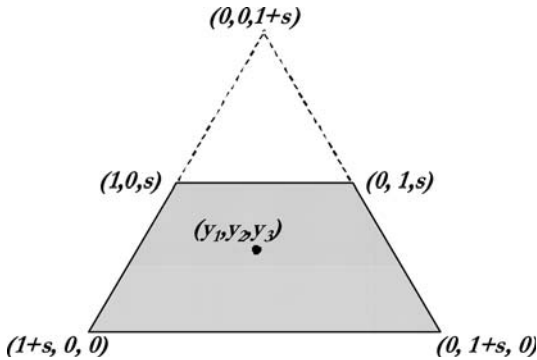
We first show that in a game where the stakeholder is active throughout the negotiations, when the stake is not too large there is a multiplicity of stationary subgame perfect equilibria. The reverse liquidity constraint of the stakeholder – her inability to obtain a share greater than her stake in any continuation – creates a slack in the equilibrium conditions that link proposals to the expected payoff of rejecting. This slack can be allocated in various ways and therefore a continuum of splits of the bilateral surplus are compatible with equilibrium. While the stakeholder does not transfer any of her stake, the range of bilateral agreement compatible with equilibrium is wide, and includes very asymmetric divisions radically different from the usual Rubinstein–Stahl shares.

Next we focus on the effects of partial participation/abstention by the stakeholder, and we show that the sheer possibility that the stakeholder may intervene in negotiations creates the potential for delays. The reason is simple: if the stakeholder does not participate in all periods, the bargainers may have an incentive to delay bilateral agreement in the hope to pressurize her into conceding extra resources. Hence, the potential participation of a stakeholder may have detrimental effects, introducing inefficiencies – in stationary strategies and under complete information – where there would not be any in its absence. The expectation of an increased surplus to share causes delays that from the point of view of the bargainers are efficient.<sup>1</sup> When all affected parties are taken into account, delays are inefficient.

One could object that since with perfect information bilateral negotiations always ensure an efficient outcome<sup>2</sup>, there is no reason why a third party, the stakeholder, should get involved. Yet, as it is often the case in services of public interest, the situation at hand may be one such that the stakeholder cannot get away from.

<sup>1</sup> This is a particular instance of the more general phenomenon discussed in Merlo and Wilson (1995).

<sup>2</sup> This is the case in the Rubinstein bargaining model. However it is well known that inefficient equilibria can obtain in alternating offers bargaining models even with complete information, once the original extensive form (Rubinstein (1982)) is modified. See chapter 7 in Muthoo (1999).



**Fig. 1** Set of feasible agreements in the stakeholder bargaining game;  $(y_1, y_2, y_3)$  is one such point

It is thus important to emphasize that the stakeholder does not choose to become involved. As she is unable to commit to inhibition and sometimes she finds herself drawn into the bargaining table. As long as the stakeholder suffers from disagreement, bargainers might succeed in extracting resources from her, and this opens the door to inefficient delays.

The present paper contributes to the literature on bargaining with arbitration or mediation.<sup>3</sup> The assumption that stakeholders enjoy a positive externality distinguishes our model from the existing literature. Our work also relates to the coalition formation literature<sup>4</sup> where externalities are a main concern, but where the role of asymmetries in the liquidity and participation constraints has not been examined so far.

Section 2 presents the model. Section 3 points out the differences with standard three player bargaining. The emergence of delays in equilibrium is addressed in section 4.

## 2 Stakeholder bargaining games

In a stakeholder bargaining (SB) game two bargainers (indexed 1 and 2) must split one unit of surplus, available only in case of agreement. A third player 3, the stakeholder, obtains a positive payoff of  $s$  if the negotiators settle their dispute, and 0 otherwise. Thus the bargainers' consensus on how to split their bilateral surplus of 1 is necessary and sufficient to generate the total surplus  $1 + s$ , where the stake  $s$  is an externality that the bilateral agreement generates for player 3. Figure 1 depicts the bargaining set in the SB game. While the set of agreements in a standard three-player bargaining problem are all points in the three-dimensional simplex of size  $1 + s$ , the feasible set of agreements in the present environment corresponds only to the shaded trapeze.

Bargaining proceeds over a (potentially infinite) number of rounds,  $t = 0, \Delta, 2\Delta \dots$ . At each period, the state of the game determines whether player 3 is active

<sup>3</sup> See Compte and Jehiel (1995), Jarque et al. (2003) and Manzini and Mariotti (2001).

<sup>4</sup> See Bloch (1996), Gomes (2003), Jehiel and Moldovanu (1995) and Montero (1999). Three player games are analysed in depth in Cornet (2003) and Gomes (2003).

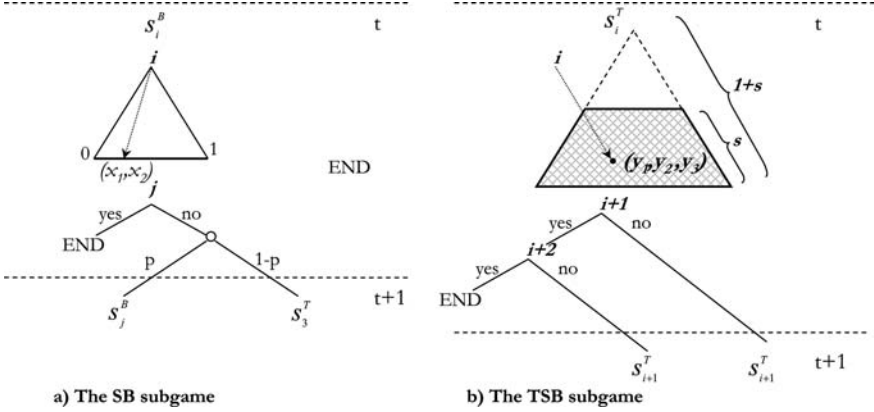


Fig. 2 Subgames

or not, specifies the set of admissible proposals and appoints the proposer. The initial state of the game is given (possibly at random). In rounds where 3 is not active the game is in a bipartite state, denoted  $s_t = s_i^B$ . All players are active in tripartite states, denoted  $s_t^T$ , where the subindex indicates the proposer. Proposals at bipartite states consists of a division of the bilateral surplus  $x = (x_1, x_2)$  with  $x_1, x_2 \geq 0$  and  $x_1 + x_2 = 1$ , and an agreement at such states yields a division of the total surplus  $(x_1, x_2, s)$ . At tripartite states, bargainers may receive resources from 3 in addition to their share of the bilateral surplus, but 3 cannot extract resources from the contending bargainers beyond the value of the stake  $s$ . Thus a proposal is a division of the total surplus  $y = (y_1, y_2, y_3)$ ,  $y_1, y_2 \geq 0$ ,  $0 \leq y_3 \leq s$  and  $y_1 + y_2 + y_3 = 1 + s$ . Each period, the proposer chooses a proposal and responders accept or reject; in tripartite states replies follow the natural order.<sup>5</sup> When a proposal is accepted (by all active responders), the game ends and the agreement is implemented. Upon rejection, play moves to the next time period and the state at  $t + 1$  is determined as follows: if  $s_t = s_i^B$  then with probability  $p$ ,  $s_{t+1} = s_j^B$ ,  $j \neq i$ , and with probability  $1 - p$ ,  $s_{t+1} = s_3^T$ ; if  $s_t = s_i^T$  then  $s_{t+1} = s_{i+1}^T$  with probability 1.<sup>6</sup> Agents have a common discount factor  $\delta = e^{-r\Delta}$ , where  $r$  is the discount rate. Consequently an agreement reached at time  $t$  allocating the total surplus according to some division  $z = (z_1, z_2, z_3)$  yields utility  $u_i(z, t) = \delta^t z_i$  to player  $i$ .

This general description includes as extreme cases pure bilateral bargaining without 3's intervention ( $s_0 = s_1^B$  and  $p = 1$ ); and tripartite stakeholder bargaining (TSB) where bipartite rounds are ruled out ( $s_0 = s_i^T$ ). The extensive form of the SB game with  $s_0 = s_i^B$  and the TSB game are sketched in Fig. 2.

<sup>5</sup> We follow the convention that 1 follows 3. More generality on the timing of replies is inconsequential. Also, we assume that at tripartite states the stakeholder has veto power over bilateral agreements, even if  $x_3 = s$ . This is only for expositional simplicity and has no consequence, since exercising this veto power is a dominated action.

<sup>6</sup> The analysis extends easily to other specifications of state transitions, including time and state dependent transition probabilities.

Strategies and subgame perfect equilibria (SPE) are defined in the standard way. Stationary strategy profiles are fully characterized by a vector specifying proposals and acceptance thresholds (whichever applicable) for each player at each state where she is active. A stationary subgame perfect equilibrium (SSPE) is a profile of strategies that constitutes an SPE. A SSPE is an *immediate agreement equilibrium* (IAE) if it yields agreement at  $t = 0$ ; and a *delayed agreement equilibrium* (DAE) is an SSPE where disagreement occurs at least in one state of the game.

We start by pointing out the fundamental difference between an SB game and a trilateral negotiation over  $1 + s$ .

### 3 The stakeholder's liquidity constraint

Assume that the initial state of the game is tripartite and negotiations begin with the stakeholder proposing at time  $t = 0$ , so that  $s_0 = s_3^T$ .

Because she cannot access any of the bilateral bargaining surplus, the stakeholder's payoff can be at most  $s$ : this produces quite dramatic effects. If  $s$  is not too great, the unique stationary equilibrium of the corresponding 'equal-rights' trilateral bargaining problem is no longer achievable; furthermore, the presence of a stakeholder introduces multiplicity of equilibria even in stationary strategies.

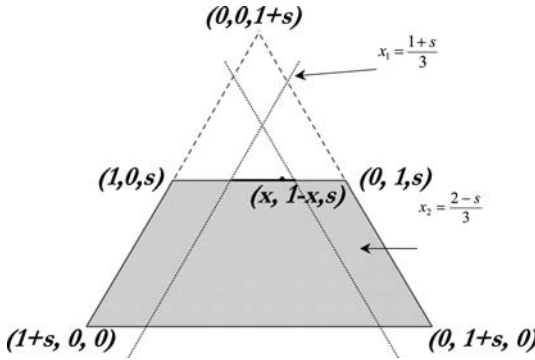
The standard trilateral alternating bargaining game over  $1 + s$  yields a unique stationary equilibrium partition, i.e.  $y^* = \left( \frac{1+s}{1+\delta+\delta^2}, \frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{\delta^2(1+s)}{1+\delta+\delta^2} \right)$  which converges to  $y^* = \left( \frac{1+s}{3}, \frac{1+s}{3}, \frac{1+s}{3} \right)$  as the discount factor approaches unity, where the first, second and third entry refer to the first, second and third mover payoffs, respectively. Because in the TSB game the stakeholder's share is bounded above by  $s$ , when  $s$  is not 'too large' there is a continuum of partitions of the bilateral surplus that can be supported in a stationary equilibrium. Moreover the splits of the bilateral surplus that are reached along such equilibria can be very different from the usual bilateral monopoly one. In the present set up if  $s$  is relatively small, when the stakeholder proposes the equilibrium payoff of the (responding) bargainers exceeds the equilibrium continuation payoff. Because of this slack, bargainers cannot take anything away from the stakeholder (whose equilibrium payoff is at any rate less than what she would get in trilateral bargaining over  $1 + s$ ). Then we can state the following:

**Proposition 1** *Assume the initial state is  $s_0 = s_3^T$  with probability 1. In the limit as  $\Delta \rightarrow 0$ :*

- (i) *When  $s$  is sufficiently small (i.e.  $s < \frac{1}{2}$ ), any proposal  $(x, 1 - x, s)$  with  $x \in \left[ \frac{1}{3}(1 + s), \frac{2-s}{3} \right]$  where the stakeholder makes no contribution can be supported in a SSPE, and there are no other SSPE.*
- (ii) *Otherwise, when  $s \geq \frac{1}{2}$ , the unique SSPE outcome coincides with that of a trilateral bargaining game over  $1 + s$ .*
- (iii) *There can be no DAE.*

*Proof* See the Appendix. □

To gain the intuition for this result, observe that the liquidity constraint on the stakeholder implies that there is a cap on the payoff that she can extract when



**Fig. 3** The equilibrium agreements of proposition 1

proposing. Fix a discount factor  $\delta \in (0, 1)$ . In any subgame, disagreement determines an overall loss in surplus equal to  $(1 + s)(1 - \delta)$ . On the other hand, when the stakeholder acts as proposer and makes no contribution, her own payoff loss in case of rejection is  $s - \delta^3 s$ , since in the continuation she would obtain  $\delta^2 s$ . Thus when the loss in overall surplus,  $(1 + s)(1 - \delta)$ , exceeds the stakeholder's loss,  $(1 - \delta^3)s$  (i.e. when  $s < \frac{\delta}{1+\delta}$ ), the stakeholder cannot retain all of this would be loss, and must distribute the 'gap' between the two bargainers. That is, when proposing, the stakeholder extracts less than she would be able to get without the liquidity constraint by holding the two bargainers to their discounted continuation payoffs. Thus the range of equilibrium agreements depends on how this gap is distributed between the two bargainers. Now consider the extreme case when the stakeholder always chooses to allocate the gap entirely to player 2. This requires an offer assigning to player 1 his discounted continuation payoff. From the point of view of the two bargainers, this is as if player 1 were given the option to put player 2 at his continuation payoff once every three periods, while player 2 is able to put his opponent down to his reservation value twice every three periods. As  $s$  vanishes, player 2 gets almost all the extra surplus, and the game approaches an alternating offer game where player 2 proposes twice as often as player 1, hence the sharing  $(\frac{1}{3}, \frac{2}{3})$ . Similar considerations apply to the case when the stakeholder allocates the whole gap to player 1, and for intermediate allocations.

The multiplicity of equilibria is a fundamental feature that does not depend on our specific formulation of the protocol. It is not difficult to check that a proposition in the same vein as Proposition 1 holds for a model with random proposers.

The set of equilibria of Proposition 1 is depicted in Fig. 3 as the thick black line, where one possible equilibrium agreement  $(x, 1 - x, s)$  is highlighted. The delimiting sloping border on the left side is the set of feasible agreements yielding a payoff of  $\frac{1}{3}(1 + s)$  for player 1. Similarly, the delimiting sloping border on the right hand side corresponds to those feasible agreements yielding a payoff  $\frac{2-s}{3}$  to player 2.

The proof of point (iii) in Proposition 1 implies that as long as all three agents are active, agreement is *not* delayed:

**Corollary 2** *There can be no DAE due to disagreement in a tripartite negotiation round.*

Details are in the Appendix, but the intuition is straightforward. In stationary strategies, the only way to obtain delayed agreement is that for some tripartite state, in equilibrium all proposals are rejected. The consequence is that the proposer in that state loses all the bargaining power deriving from her role as proposer. On the other hand, all of the agents ‘pay’ the cost of a rejection by moving negotiations to the next round. This is however what destroys the possibility of delay in equilibrium, as the agent whose offer is always rejected can ‘bank’ on these costs and make an offer which makes her better off and such that responders have an incentive to accept. To see this, take for instance a candidate equilibrium where the stakeholder’s proposal is always rejected. In this case the bargainers can coordinate and extract the entire stakeholder’s surplus, by insisting on a complete handout each time they make a proposal. The stakeholder would have no incentive to refuse such a proposal. A rejection would trigger either a subgame where the stakeholder is a proposer, in which case no agreement follows; or one in which one of the bargainers makes the same offer the stakeholder has just rejected. However, precisely for this reason, this cannot be an equilibrium, for the stakeholder has an incentive to make an offer that would be accepted. For instance, she could offer immediately the same surplus division as a bargainer would in the next round, and give up all of her stake but a small amount. Similar arguments apply to the candidate equilibrium where a bargainer’s proposal is always rejected. It is then immediate that TSB games, since all agents are active at all rounds, cannot have DAE.

**A remark on delay and non-stationary equilibria:** In multilateral bargaining games of alternating proposals, stationary equilibria do not exhaust the set of SPE. Other non-stationary equilibria may exist<sup>7</sup>. This is the case in the present model, too. Non-stationarity allows us to construct equilibria with delayed agreement. At any rate, in general, the punishments needed to support delayed equilibria of this type are rather extreme and seldom observed in practice.<sup>8</sup>

#### 4 Limited participation and delays

We now turn our attention to games where the stakeholder’s participation is not always assured. We show that this translates in the possibility that in a stationary equilibrium agreements are delayed. As discussed above, if a DAE exists, it must be that the stakeholder is not always active, and some rounds of play take place in a bipartite state where bargainers disagree. Thus the question is whether the stakeholder’s limited participation is in itself sufficient for a DAE to exist, and if so, under what conditions are efficient equilibria obtained.

As we show below, in a SB game that combines bipartite states and some tripartite states (where the stakeholder acts as responder), when the stake is sufficiently sizeable, the unique SSPE is a DAE. The intuition for this result is that whenever the stakeholder is involved in negotiations as a responder, the possibility arises for the bargainers to extract a sizable part of her stake. Thus, because bargainers can extract additional surplus from the stakeholder, if the stake is sufficiently large they have the incentive to ‘sit it out’, that is, to disagree in bipartite states and wait for

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<sup>7</sup> See e.g. section 3.13 in Osborne and Rubinstein (1990).

<sup>8</sup> Complexity arguments can be used to justify stationary strategies in bargaining games. See for instance Sabourian and Chatterjee (2000).

the stakeholder to become active. On the other hand, when the stake is sufficiently small, the unique equilibrium is an IAE:

**Proposition 3** *For each SB game with  $s_0 = s_i^B$  and  $0 < p < 1$ , in the limit as  $\Delta \rightarrow 0$ :*

- (i) *if  $s \geq \frac{1}{2}$  the unique equilibrium is a DAE in which disagreement prevails at all  $s_t = s_i^B$  and immediate agreement prevails at  $s_t = s_i^T$ ;*
- (ii) *if  $s < \frac{1}{2}$  the unique equilibrium is an IAE.*

*Proof* See the Appendix. □

To see why Proposition 3 holds, recall that Corollary 2 establishes that there can be no disagreement in tripartite states. Consequently, a DAE requires disagreement in bipartite states. This can be supported in equilibrium only if the expected discounted continuation joint payoff exceeds the joint payoff from a bilateral agreement. A necessary condition is that some contribution from the stakeholder is forthcoming, which is possible, as we saw above, only when the stake is sufficiently large.

## 5 Conclusions

We have introduced stakeholder bargaining games as a novel frame of reference for bilateral negotiations when the interests of third parties – stakeholders – are affected, so that – and this is the crucial feature – the stakeholder may have an interest to inject additional resources in the dispute in order to encourage agreement. Our analysis makes two main points: (1) When the participation of the stakeholder is assured, a continuum of equilibria exist provided that the stake is not too great, and very asymmetric bilateral agreements may prevail. (2) When the participation of the stakeholder is uncertain inefficiencies arise.

Besides being suited to analyze negotiations which are originally configured as stakeholder bargaining games, our model lends itself to a number of possible extensions. For instance it could be used to investigate the incentives that exist in bilateral bargaining to relinquish irrevocably one's bargaining rights and thereby opt for a decentralized bargaining protocol. For concreteness, a public authority with the ability to relocate property rights may want to separate into two (a stakeholder and a bargainer) when negotiating with another party. Our analysis suggests that governments may have an incentive to decentralize – and become stakeholders rather than directly involve negotiating partners – as they may gain as stakeholders an advantage which is lost in direct negotiations. A closely related issue is the extent to which a stakeholder might *choose* to intervene in negotiations.<sup>9</sup> Finally, a more realistic approach would consider the effects of uncertainty over the benefits and resources available to the stakeholder. We leave these issues to future research.

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<sup>9</sup> In Ponsati (2004) and Manzini and Ponsati (2005), we explore games where the stakeholder's decision to participate is strategic, modelling the environment as a concession game.



## Appendix

### *Proof of Proposition 1*

(i) Let  $x^i = (x_1^i, x_2^i, x_3^i)$  denote the surplus division of  $1 + s$  proposed by agent  $i$ , where  $x_1^i + x_2^i + x_3^i = 1 + s$  and  $x_3^i \leq s$ . Assume that  $x^3 = (x, 1 - x, s)$ . With  $x \in \left[ \frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{1+\delta-\delta^2s}{1+\delta+\delta^2} \right]$  and  $\delta \in (0, 1)$ , equilibrium strategies are for 1 to propose  $x^1$  with  $x_1^1 = \delta^2x + (1 - \delta)(1 + s)$  and  $x_3^1 = (1 - \delta^2)s$ ; for 2 to propose  $x^2$  with  $x_2^2 = 1 - \delta x + (1 - \delta)s$  and  $x_3^2 = (1 - \delta)s$ ; and for the stakeholder to propose  $(x, 1 - x, s)$ . Regarding responses, players accept offers yielding at least the present discounted value of the equilibrium continuation payoff and reject otherwise. Equilibrium proposals are derived based on the following considerations.

A proposal is accepted by the stakeholder in a subgame where bargainer  $i$  proposes if

$$\begin{aligned} x_3^2 &= \delta s \\ x_3^1 &= \delta^2 s. \end{aligned}$$

Bargainer  $i$ 's proposed share  $x_i^i$  for herself must satisfy

$$\begin{aligned} 1 + (1 - \delta^2)s - x_1^1 &= \delta x_2^2 \\ 1 + (1 - \delta)s - x_2^2 &= \delta x. \end{aligned}$$

Given their strategies, the stakeholders' proposal  $(x, 1 - x, s)$  is acceptable to the bargainers only if

$$\begin{aligned} x &\geq \delta x_1^1 = \delta (\delta^2 x + (1 - \delta)(1 + s)) \Rightarrow x \geq \frac{\delta(1 + s)}{1 + \delta + \delta^2} \equiv \underline{x} \\ 1 - x &\geq \delta x_2^2 = \delta (1 + s - x_1^1 - \delta^2 s) = \delta^2 (1 - \delta x + (1 - \delta)s) = \delta^2 x_2^2 \\ &\Rightarrow x \leq \frac{1 + \delta - \delta^2 s}{1 + \delta + \delta^2} \equiv \bar{x} \end{aligned}$$

whereby

$$\bar{x} > \underline{x} \Leftrightarrow s < \frac{1}{\delta(1 + \delta)} = \underline{s}.$$

From the equilibrium conditions above it is clear that only proposals  $(x, 1 - x, s)$  with  $x \in \left[ \frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{1+\delta-\delta^2s}{1+\delta+\delta^2} \right]$  can be supported in an SSPE. Moreover, at least one bargainer obtains a payoff in excess of his continuation payoff. If there were another SSPE, then it would have to be the case that the stakeholder makes a contribution  $\gamma_i \geq 0$  to each bargainer,  $0 < \gamma = \gamma_1 + \gamma_2 \leq s$ , so that the outcome is  $(x + \gamma_1, 1 - x + \gamma_2, s - \gamma)$ , and at least one bargainer obtains a payoff in excess of his continuation payoff. Both players getting a payoff in excess of their continuation value cannot be an equilibrium, since the stakeholder could decrease her contribution a bit and still get acceptance.

Say that only 2 gets more than his continuation value. Then the stakeholder could improve on this offer by a proposal  $(x + \gamma'_1, 1 - x + \gamma'_2, s - \gamma')$  with  $\gamma'_1 = \gamma_1 + \varepsilon$  and  $\gamma'_2 = \gamma_2 - 2\varepsilon$ : if  $\varepsilon$  is small enough, both bargainers will still want to accept, and the stakeholder will retain a higher portion of her payoff. Taking the limit as  $\Delta \rightarrow 0$  (so that  $\delta \rightarrow 1$ ) proves the claim.

- (ii) The proof is standard, thus omitted for the sake of brevity, see e.g. Osborne and Rubinstein (1990).
- (iii) Consider state  $s_i^T$  and let  $V_j$ , for  $j = 1, 2, 3$ , denote players' ex-ante expected payoffs in the continuation after disagreement prevails in this state. In any DAE,  $V_1 + V_2 + V_3 \leq 1 + s$ , so that  $1 + s > \delta(1 + s) \geq \delta(V_1 + V_2 + V_3)$ . The two extremes of this last set of inequalities guarantee that the proposer,  $i$ , is better off by having her proposal accepted, as a deviation to an acceptable proposal (yielding responders  $j, k \neq i$  a total share  $\delta(V_j + V_k)$ ) guarantees a higher payoff than any proposal that  $j$  and  $k$  reject (since  $1 + s - \delta(V_j + V_k) > \delta V_i$ ). Note that when the proposer is the stakeholder, either  $s = \max\{s, 1 + s - \delta(V_1, V_2)\}$ , so that the previous argument applies; or  $s \neq \max\{s, 1 + s - \delta(V_1, V_2)\}$ , so that by deviating to an acceptable offer the stakeholder would achieve a payoff of  $s$ , which exceeds any discounted continuation payoff.  $\square$

*Proof of Proposition 3* Recall from the proof of Proposition 1 that  $\underline{s} \equiv \frac{1}{\delta(1+\delta)}$ . From Corollary 2, a DAE requires that (equilibrium) proposals in bipartite states are rejected. Denote by  $W_i^B = V_{i1}^B + V_{i2}^B$  the joint expected payoffs of bargainers upon disagreement at state  $s_i^B$ , and by  $W^T = V_1^T + V_2^T$  the joint expected payoff in tripartite states  $s_3^T$ . Obviously  $W_i^B$  must solve  $W_i^B = p\delta W_j^B + \delta(1-p)W^T$ . The continuation payoff to the bargainers in tripartite states  $s_3^T$  depends on how large the stake  $s$  is. Thus there can be two cases:

(A) Suppose  $s \geq \underline{s}$ , so that the outcome in state  $s_3^T$  is immediate agreement at the standard trilateral partition  $\left(\frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{\delta^2(1+s)}{1+\delta+\delta^2}, \frac{1+s}{1+\delta+\delta^2}\right)$ . In this case,  $W_i^B$  solves  $W_i^B = p\delta W_j^B + \delta(1-p)\frac{(\delta+\delta^2)(1+s)}{1+\delta+\delta^2}$ , so that  $W_i^B = W_j^B = \delta^2\frac{(1+s)(1+\delta)(1-p)}{(1+\delta+\delta^2)(1-p\delta)} \equiv W^B$ . Then, in bipartite states, disagreement prevails when  $\delta W^B > 1$ , and this inequality holds provided that  $\delta^3\frac{(1+s)(1+\delta)(1-p)}{(1+\delta+\delta^2)(1-p\delta)} > 1$ , or equivalently for  $s > \hat{s}$  (case (i) in Proposition 3). If instead  $s \leq \hat{s}$ , then  $\delta W^B \leq 1$ , so that the unique stationary equilibrium is with immediate agreement at all  $s_i = s_i^B$  (case (ii) in Proposition 3). Note that  $\hat{s} \geq \underline{s}$ .<sup>10</sup>

(B) Suppose  $s < \underline{s}$ , so that the outcome in state  $s_3^T$  is an immediate agreement on a partition in which the stakeholder makes no contribution. Then the bargainers joint payoff in state  $s_3^T$  is 1, and there are no incentives for delay, thus agreement in the bipartite state is immediate. Supporting strategies are the obvious ones (i.e. in DAE bargainers make unacceptable offers and in tripartite states follow the strategies which support the equilibria of Proposition 1, while in IAE proposers make the just acceptable offers, that is the expected present discounted value of the equilibrium continuation payoff in case of disagreement). Taking the limit as  $\Delta \rightarrow 0$  (so that  $\delta \rightarrow 1$ ) yields the statement of the proposition.  $\square$

<sup>10</sup> In fact  $\hat{s} - \underline{s} = \frac{\delta(1-\delta)(1+\delta)^2 + 1 - p\delta(1+\delta-\delta^3)}{\delta^3(1+\delta)(1-p)} - \frac{1}{\delta(1+\delta)} = (1-\delta)\left(1 + \delta + \delta^2\right)\frac{1+\delta(1-p)}{\delta^3(1+\delta)(1-p)} > 0$ .

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