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Stakeholders in bilateral conflict

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Abstract

The resolution of a conflict often has an impact which extends beyond the remits of the parties directly involved in the confrontation (e.g. labour negotiations in sectors of public interest, where a strike would impact on the public at large). Once this is recognised, models addressing negotiations in such situations ought to account for the role and interests of the stakeholder—a third party whose stake is linked to the original negotiations. In this paper we address the strategic role of stakeholders in bilateral confrontations that take the form of a war of attrition; we assume that the bilateral confrontation runs concurrently with the parties interaction with the stakeholder, that chooses strategically her timing to intervene and take action to promote agreement. We show that under complete information the interplay of different interests in this tripartite timing game results in delayed outcomes. We also explore the role of incomplete information and show that asymmetries of information do not necessarily translate in increased inefficiency.

Keywords: Stakeholders; Bargaining; War of attrition

JEL classification: C7; D74; J5

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1. Introduction

Stalemate and conflict in negotiations often produce considerable and direct effects on agents other than the prime sides to the dispute. For instance, whenever industrial action looms on negotiations in matters of public concern, such as transport, health, the supply of utilities, governments have a vested interest in a speedy resolution of the dispute. Thus negotiations over wages, layoffs, working conditions and so on extend well beyond the concerns of the parties directly involved; the consequence is that the third party which is unwillingly pulled into negotiations has a concrete interest to put up resources and protect his own stake in the dispute. Our objective in this paper is to study the role of an interested third party in bilateral conflicts within a model that emphasizes tripartite interactions in the timing of mutual concessions.

We analyze the role of stakeholders in bilateral conflicts that take the form of a war of attrition—a game where parties choose a time to concede to each other's demands. Our main emphasis in this paper is on the timing of agent's mutual concessions and their concurrent interaction with the timing of stakeholder's actions to promote agreement.[1] To begin with, we consider a perfect information setup–i.e. circumstances where the issues under negotiation are straightforward and negotiating stances well publicized–and we show that protracted stalemate can occur even so. Next we turn to instances where negotiators lack an accurate assessment of the means at the stakeholder's disposal and where the stakeholder is uncertain about the surplus available in the bilateral negotiation. We show that under such incomplete information, increasing the degree of uncertainty that agents face does not necessarily translate in increased inefficiency. Although the possibility of delay can never be ruled out altogether, an immediate agreement occurs with substantial probability under a 'large' degree of uncertainty, and it becomes impossible when the uncertainty decreases.

To get a flavor of our results, consider the complete information setup first. The presence of a stakeholder (e.g. the government) implies that the original set of negotiations runs in parallel with negotiations involving the stakeholder. The crucial issue is that the resources bargained upon on the two negotiating tables are *interconnected*, the terms of each agreement are conditioned by the outcome of the other one. The resolution of the 'meta'-dispute which encompasses all parties hinges upon the resolution of each stalemate: this creates the potential for a strategic interplay between the two sets of negotiations (the original one and the one with the stakeholder), which may be the cause of severe inefficiency, even in the absence of informational asymmetries.

The reason is easy to illustrate with an example. Consider a firm threatening to close a plant during an economic downturn. The impact that job losses may have on the economy at large and on voter behavior might induce the involvement of the government, that could intervene on the one hand with a handout to the firm, and on the other with financial support for re-training, income support, etc. to the employees' side. So two deals need to

[1]In Manzini and Ponsatí (2001) we examine the role of stakeholders in bilateral bargaining games á la Rubinstein, emphasizing the effect of (potential) transfers from the stakeholder to the bargainers. The impact of transfers (aid) over agents engaged in a bilateral war of attrition is analyzed in Ponsatí (2004).

be stricken, one between the government and the workforce, and one between the firm and the government. The central feature is that to concede over the terms of employment to the workforce, the government will bear in mind the development of its negotiations with the firm. In this setting the government can single-handedly draw the dispute to a speedy conclusion, by conceding on both negotiating tables; or impose a drawn out process to both of the other sides. Provided the gains from obtaining its most favored alternative are sufficiently high, the latter may be an attractive prospect for the government.

This sort of inefficiency could be removed if all parties were involved in negotiations over all issues, but this is often unfeasible or undesirable for some of the parties.[2] Alternatively, one could argue that inefficiencies may be removed if governments strengthened their reliance on legislation. In fact, though, the tendency of modern governments is towards less and less mandatory intervention, with the consequence that governments become progressively more active players in negotiations, rather than 'referees'.[3]

In what follows Section 2 discusses interlinked negotiations under complete information and Section 3 explores the effect of asymmetric information.

2. Complete information

The government is engaged in two sets of negotiations over financial support, one with the employees, and one with the firm. All agents discount utility at the instantaneous rate r_i (with i=f, u, g for firm, worker's union and government, respectively), so that an agreement reached at time t over a payment x to agent i yields to this agent a utility of $xe^{-r_i t}$ in present discounted value. Agents obtain their payoff only when both sets of negotiations are over. The overall government stake in negotiations is S, part of which can be used to settle the union's side and part of which the firm's side of the dispute.

In negotiations between the firm and the union, let w_u be the workers' preferred settlement, and $w_g < w_u$ the wage preferred by the government. If agent $i = u, g$ concedes at time t_i, then this implies agreement on wage $w_{j \neq i}$. If both agents concede at the same time, then each of the two agreements is implemented with equal probability. The time when an agreement is enjoyed depends on whether or not the parallel set of negotiations (between the firm and the government) is over or not. If it is, then the wage settlement is implemented immediately upon agreement. If not, then the wage settlement is implemented as soon as agreement is reached in the firm–government negotiations. If we denote by $\tau = \min\{\tau_g, \tau_f\}$ the time at which an agreement is reached in firm–

[2]Moreover, inefficient equilibria might be pervasive in multilateral negotiations (see, e.g. Osborne and Rubinstein (1990) and references therein).

[3]For example, in 2002 in the UK the then Secretary of State for the Department of Trade and Industry Byers declared "Government needs to be active but must not be interventionist. In doing so I reject the approaches of the past. That of the new right who took the line that the market should be the 'be all and end all' and that government should simply keep out. And that of the old left who believed in large scale intervention, coupled with massive state subsidies which either sought to back winners or to rescue failing companies." See Institute of Public Policy Research (2000).

government bargaining, the union's payoff in wage negotiations, denoted by $v_u(t_g, t_u; \tau)$, can be written as:

$$v_u(t_g, t_u; \tau) = \begin{cases} w_i e^{-r_u t_j} & \text{if } \tau \leq t_j < t_i \text{ for all } i,j = g,u \text{ and } i \neq j; \\ \frac{w_u + w_g}{2} e^{-r_u \max\{t,\tau\}} & \text{if } t_g = t_u = t; \\ w_i e^{-r_u \tau} & \text{if } t_j \leq \min\{\tau, t_i\} \text{ for all } i,j = g,u \text{ and } i \neq j. \end{cases} \quad (1)$$

where the top (bottom) row refers to payoffs when wage negotiations terminate with a single concession after (resp., before) the time τ of agreement in firm–government negotiations, and the middle row considers those cases where the union and the government concede at the same time (either before or after the other set of negotiations has terminated, depending on whether $t \gtreqless \tau$).

The concurrent set of negotiations between the firm and the government develops in similar fashion. Let h_g be the handout to the firm favored by the government, and $h_f > h_g$ the outcome preferred by the firm. If agent $i = g,f$ concedes at time τ_i, this means that an agreement is struck on government payment $h_{j \neq i}$. If both agents concede at the same time, then each of the two agreements is implemented with probability of (1/2). As above, if there is already agreement in the government–union wage negotiations, then the settlement over the government's payout to the firm is implemented immediately. Alternatively, it is implemented as soon as an agreement is reached over wages. Let $t = \min\{t_u, t_g\}$ denote the time at which an agreement is reached in the parallel set of negotiations between the government and the union. The payoff for the firm, denoted by $v_f(\tau_f, \tau_g; t)$, is:

$$v_f(\tau_f, \tau_g; t) = \begin{cases} h_p e^{-r_u \tau_q} & \text{if } t \leq \tau_q < \tau_p \text{ for all } p,q = g,f \text{ and } p \neq q; \\ \frac{h_g + h_f}{2} e^{-r_f \max\{t,\tau\}} & \text{if } \tau_g = \tau_f = \tau; \\ h_q e^{-r_f t} & \text{if } \tau_p \leq \min\{t, \tau_q\} \text{ for all } p,q = g,f \text{ and } p \neq q. \end{cases} \quad (2)$$

The government's payoff $v_g(t_g, t_u, \tau_f, \tau_g)$ is

$$v_g(t_g, t_u, \tau_f; \tau_g) = \begin{cases} (S - h_p - w_j) e^{-r_g \max\{t_i, \tau_q\}} & \text{if } \tau_q < \tau_p \text{ and } t_i < t_j; \\ \left(S - h_p - \frac{w_g + w_u}{2}\right) e^{-r_g \max\{t, \tau_q\}} & \text{if } \tau_q < \tau_p \text{ and } t_g = t_u = t; \\ \left(S - \frac{h_g + h_f}{2} - w_j\right) e^{-r_g \max\{t_i, \tau\}} & \text{if } \tau_g = \tau_f = \tau \text{ and } t_i < t_j; \\ \left(S - \frac{h_g + h_f + w_g + w_u}{2}\right) e^{-r_g \max\{t, \tau\}} & \text{if } \tau_q = \tau_p = \tau \text{ and } t_g = t_u = t; \end{cases} \quad (3)$$

for all $i,j = g,u$ with $i \neq j$ and for all $p,q = g,f$ with $p \neq q$.

As a preliminary it is useful to define τ_f^* and t_u^* as the times such that in negotiations with the government the firm and the union, respectively, are indifferent between (i) holding out and obtaining the preferred proposal with delay, and (ii) conceding immediately to the government's proposal. Then τ_f^* solves $h_f e^{-r_f \tau} = h_g$ and t_u^* solves $w_u e^{-r_u t} = w_g$, yielding

$$\tau_f^* = \frac{1}{r_f} \ln\left(\frac{h_f}{h_g}\right) > 0, \quad t_u^* = \frac{1}{r_u} \ln\left(\frac{w_u}{w_g}\right) > 0. \quad (4)$$

Similarly, define for the government the threshold stopping time $t_{\mathrm{g}}^{*}=\frac{1}{r_{\mathrm{g}}}\ln\left(\frac{S-w_{\mathrm{g}}-h_{\mathrm{g}}}{S-w_{\mathrm{u}}-h_{\mathrm{f}}}\right)$ which makes the government indifferent between conceding immediately and holding out in both sets of negotiations, that is $(S-w_{\mathrm{g}}-h_{\mathrm{g}})\mathrm{e}^{-r_{\mathrm{g}}t_{\mathrm{g}}^{*}}=(S-w_{\mathrm{u}}-h_{\mathrm{f}})$.

Note that at any time before these threshold times τ_{f}^{*}, t_{g}^{*} and t_{u}^{*}, the agent prefers to hold on rather than concede, since the payoff if conceding is less than the payoff if the opponent concedes at the threshold, that is $h_{\mathrm{f}}\mathrm{e}^{-r_{\mathrm{f}}\tau}\geq h_{\mathrm{g}}\Leftrightarrow\tau\leq\tau_{\mathrm{f}}^{*}$; $w_{\mathrm{u}}\mathrm{e}^{-r_{w}t}\geq w_{\mathrm{g}}\Leftrightarrow t\leq t_{\mathrm{u}}^{*}$; and $(S-w_{\mathrm{g}}-h_{\mathrm{g}})\mathrm{e}^{-r_{\mathrm{g}}t}\geq(S-w_{\mathrm{u}}-h_{\mathrm{f}})\Leftrightarrow t\leq t_{\mathrm{g}}^{*}$.

A pure strategy for either the union or the firm is simply a concession time, i.e. $t_{\mathrm{u}},\tau_{\mathrm{f}}\in R_{+}$; a pure strategy for the government is instead a pair of concession times, one for each war of attrition in which it is involved, i.e. $(t_{\mathrm{g}},\tau_{\mathrm{g}})\in R_{+}^{2}$. Consequently a pure strategy profile is a vector $S=(t_{\mathrm{u}},\ \tau_{\mathrm{f}},\ (t_{\mathrm{g}},\ \tau_{\mathrm{g}}))$ of concession times.

2.1. Delayed equilibria

It is immediate to verify that, as in the standard war of attrition, there are subgame perfect equilibria (henceforth s.p.e.) with immediate agreement. In all these equilibria one agent backs down immediately, giving in to the (credible) threat that his opponent will hold on for long enough (i.e. beyond the threshold t_{i}^{*} of the conceding agent i). It is however more interesting to consider whether in this simple setup delayed equilibria can arise in pure strategies. This can occur only if both sets of negotiations are delayed; furthermore, as the next lemma establishes, in any delayed equilibrium both negotiations must end at the same time. For suppose not, and assume $\tau<t$. Then the conceding player in wage negotiations has a profitable deviation, since anticipating concession to $t'=\tau$ leads to the same agreement earlier. Similarly for the case $t<\tau$. Thus we can state:

Lemma 1. *In any delayed s.p.e. it must be* $\tau=t$.

Lemma 1 highlights the distinctive feature of any delayed equilibrium, which is that the government uses strategically its participation in both negotiations, exploiting the possibility to impose single-handedly delays to both the other negotiators. As long as this threat is credible, the other side in each negotiation will concede. Consequently, the government never concedes in both negotiations. This is stated more formally in the following proposition, which establishes that in equilibrium either the government hold out in handout negotiations with the firm while conceding in wage negotiations, or in wage negotiations while conceding to the firm, or in both:

Proposition 1. *Let* $w_{u}>w_{g}\geq0$ *and* $0\leq h_{g}<h_{f}$. *Define* $\bar{t}_{\mathrm{L}}=\frac{1}{r_{\mathrm{g}}}\ln\left(\frac{S-w_{\mathrm{u}}-h_{\mathrm{g}}}{S-w_{\mathrm{u}}-h_{\mathrm{f}}}\right)$, $\bar{t}_{\mathrm{R}}=\frac{1}{r_{\mathrm{g}}}\ln\left(\frac{S-w_{\mathrm{g}}-h_{\mathrm{f}}}{S-w_{\mathrm{u}}-h_{\mathrm{f}}}\right)$ and $\bar{t}_{\mathrm{I}}=\frac{1}{r_{\mathrm{g}}}\ln\left(\frac{S-w_{\mathrm{g}}-h_{\mathrm{g}}}{S-w_{\mathrm{u}}-h_{\mathrm{f}}}\right)$. *Then any delayed pure strategy s.p.e. will be of one of the following types:*

1. *Concession over Wage (CW): for any time* $t\in(0,\ \bar{t}_{L})$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time t.*
2. *Concession over Handout (CH): for any time* $t\in(0,\bar{t}_{R})$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time t.*

3. *No Concessions (NC): for any time $t \in (0, \bar{t}_I)$ there exists a pure strategy s.p.e. where the government holds out in both wage and handout negotiations, and where agreement is reached at time t.*

Proof. See Appendix A.

The supporting strategies are specified in Appendix A: the common feature for all equilibria is that the firm and the union are at a disadvantage in negotiations with the government, which always prevails in at least one of the two negotiations. No unilateral deviation by either the firm or the union can impact on the overall outcome of negotiations: an earlier concession would not anticipate the overall end of negotiations. On the other hand, a deviation to holding on even further would not be profitable: although this may mean obtaining the most preferred settlement, this would come at a time which is late enough to make this proposition unappealing. To the contrary, unilateral deviations by the government can indeed change the implementation date for all agreements, and it is this which gives the government a stronger bargaining position.

Note that the maximum delay can be quite lengthy, as it depends on the level of conflict, that is on the difference between w_g and w_u, and between h_g and h_f; furthermore, the more the union and the firm's claims eat into the government's stake (so that $S - w_u - h_f$, the denominator for $\bar{t}_L$, $\bar{t}_R$ and $\bar{t}_I$), and the more patient the government is (i.e. the lower r_g), the longer the potential delay for agreement. Intuitively, the more the government has to lose by conceding, the more it pays to hold out and force the preferred settlement.

2.2. *Incompatible demands?*

So far we have assumed that negotiators start at the outset with incompatible demands. Although this is not an unreasonable assumption, it is worth investigating whether these demands can be endogenized in some 'pre-war of attrition' stage, and what the impact is on the behavior in the war of attrition, which we do here.

Each of the two negotiations (the one over wages and the one over the government handout) is thus composed of two phases. In the bargaining phase each of the two sides in the negotiations tables its proposal. If demands are compatible, then an agreement is reached. Alternatively, negotiations enter the war of attrition phase described in Section 2 and agents obtain their payoff only when both sets of negotiations are over. The bargaining phase is structured as a simple Nash demand game, where both sides involved table their claims/offers independently. Proposed wage settlements w_i and government contributions h_i are allowed to vary within some bounded interval, i.e. $w_i \in [\underline{w}, \overline{w}]$ and $h_i \in [\underline{h}, \overline{h}]$, with $\underline{w} \leq \overline{w}$ and $\underline{h} \leq \overline{h}$. For instance, there may be either budgetary or legal constraints to how much the stakeholder can contribute to support the firm, or there may be a minimum wage in operation.

In wage negotiations, w_g and w_u now denote the wage settlements put forward by the government and the worker's union, respectively. If $w_g \geq w_u$, then these wages are compatible, and this set of negotiations ends in agreement, with the union accepting w_g. If instead $w_g < w_u$, then negotiations enter the war of attrition stage described above.

Negotiations between the firm and the government develop in similar fashion. Now h_g is the handout to the firm proposed by the government, and h_f that claimed by the firm. If

$h_g \geq h_f$, negotiations end with the firm obtaining h_g. If instead $h_g < h_f$, negotiations enter the war of attrition stage.

Obviously this setup allows subgame perfect equilibria in which initial demands are compatible, so that agreement is reached immediately. A more interesting issue is to examine whether the inefficient (delayed) equilibria of the base model are wiped out by the introduction of the first stage, in which demands are chosen strategically. As we show below, the strategic incentives to delay agreement are still paramount, and delayed equilibria still occur in pure strategies. As a preliminary note that a strategy for a player now has to specify an initial claim and a concession time, so that a strategy profile is now a vector $\tilde{S}=((w_u, t_u),(h_f, \tau_f),(w_g, h_g, t_g, \tau_g))$.[4] As the next proposition establishes, even when demands are endogenized the same pattern of delayed agreements as with exogenous demands can be supported in a perfect equilibrium. There are three classes of delayed equilibria, one where the government is sympathetic to the union, and eventually concedes to its claim while resisting requests from the firm ('leftwing government equilibrium'); one where the government holds firm with the union but concedes the maximum payoff to the employer ('rightwing government equilibrium'); one in which the government does not concede to any party ('intransigent government equilibrium'):

Proposition 2. *Let* $w \in (\underline{w}, \overline{w})$, $h \in (\underline{h}, \overline{h})$, *and define* $\bar{\tau}_R = \frac{1}{r_g} \ln\left(\frac{S-\underline{w}-\overline{h}}{S-w-\overline{h}}\right)$, $\bar{\tau}_L = \frac{1}{r_g} \ln\left(\frac{S-\overline{w}-\underline{h}}{S-\overline{w}-h}\right)$, $\bar{\tau}_I = \frac{1}{r_g} \ln\left(\frac{S-\underline{w}-\underline{h}}{S-w-h}\right)$. *then:*

1. *'Leftwing government' equilibrium (LGE): For any time* $\tau \in (0, \bar{\tau}_L)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* τ.
2. *'Rightwing government' equilibrium (RGE): For any time* $\tau \in (0, \bar{\tau}_R)$ *there exists a pure strategy s.p.e. where the government concedes in wage negotiations but holds out in handout negotiations, and where agreement is reached at time* τ.
3. *'Intransigent government' equilibrium (IGE): For any time* $\tau \in (0, \bar{\tau}_I)$ *there exists a pure strategy s.p.e. where the government holds out in both wage and handout negotiations, and where agreement is reached at time* τ.

Proof. See Appendix A.

Thus delays can still occur even when agents can formulate their initial demands: it pays to risk delay by setting as high a claim as possible if the claim is eventually successful.

3. Incomplete information

In this section we model the situation where the government is uncertain about the need for public intervention while the union and the firm ignore the value of the government's stake. For simplicity, we assume that the firm–union pair faces no uncertainty about their

[4] Here we abuse notation for ease of exposition, as obviously the timing of concession is going to depend on the demands made in the first stage—see Appendix A for details.

bilateral surplus, and we treat their joint decisions as those of a single player to which we refer as 'the bargainers'. Consequently the interaction among the three agents is modelled as a game of incomplete information between two players only, the bargainers and the government.

Formally, the bilateral surplus between the bargainers is $1-c_{\mathrm{b}}$ and the government stake is $G-c_{\mathrm{g}}$, where c_{b} and c_{g} are random variables uniformly distributed, respectively, in $[0, C_{\mathrm{b}}]$, $C_{\mathrm{b}}>1$ and $[0, C_{\mathrm{g}}]$, $C_{\mathrm{g}}>G$. The government privately observe the realization of c_{g} while the realization of c_{b} is privately observed by the bargainers.

The game between the bargainers and the government is again a timing game: each side can terminate the game at any $t\in[0, \infty)$. Bargainers terminate by reaching a bilateral agreement without government intervention. Upon such termination at date t, the bargainers split $1-c_{\mathrm{b}}$–each obtains $\frac{1-c_{\mathrm{b}}}{2}\mathrm{e}^{-t}$–and the government receives $(G-c_{\mathrm{g}})\mathrm{e}^{-t}$. Alternatively the bargainers can delay, that is, they may remain in disagreement hoping that the government will intervene. On its side, the government may decide to intervene at any t, or simply delay and wait for a bilateral agreement. The government decision to intervene at t is a terminal move as well because, if the trilateral negotiation ensues, all private information is revealed and the three parties reach an immediate agreement dividing the total surplus equally so that each agent receives $\frac{G+1-c_{\mathrm{b}}-c_{\mathrm{g}}}{3}\mathrm{e}^{-t}$.

A *Strategy* of player i, i=b,g is a function t^i selecting, for each $c_i\in[0, C_i]$, the date $t^i(c_i)$ at which player i of type c_i terminates the game. Let $V^i(c_i, t, t^j)$ denote the expected gains from termination at t for type c_i, given the opponent's strategy t^j. The focus of our attention will be pairs of strategies that constitute a (Bayesian) Equilibrium; that is $(t^{\mathrm{g}}, t^{\mathrm{b}})$ such that

$$t^{\mathrm{g}}(c_{\mathrm{g}}) = \arg\max V^{\mathrm{g}}(c_{\mathrm{g}}, t, t^{\mathrm{b}})$$

$$t^{\mathrm{b}}(c_{\mathrm{b}}) = \arg\max V^{\mathrm{b}}(c_{\mathrm{b}}, t, t^{\mathrm{g}}), \tag{5}$$

for all $c_i\in[0, C_i]$.[5] We limit our attention to equilibria in strategies that are differentiable in t a.e.[6]

So what happens along the equilibrium path? Three observations are in order.

1. Equilibrium strategies must be type-monotone; that is, $t^i(c_i)\leq t^i(c_i')$ for all $c_i<c_i'$, where the inequality is strict unless $t^i(c_i)=0$. This result is standard and its intuition straightforward: the larger the surplus, the larger the opportunity cost in case of disagreement, so that a concession looks more attractive than haggling. Type-monotonicity implies our next observation.
2. An equilibrium strategy profile $(t^{\mathrm{g}}, t^{\mathrm{b}})$ is fully characterized by strictly increasing functions $(g(.), b(.))$–the inverses of $(t^{\mathrm{s}}, t^{\mathrm{b}})$–mapping dates into types such that $(g(t), b(t))=(c_{\mathrm{g}}, c_{\mathrm{b}})$ if and only if $t^{\mathrm{g}}(c_{\mathrm{g}})=t$ and $t^{\mathrm{b}}(c_{\mathrm{b}})=t$.

[5]Observe that, in the present informational environment, Perfect Bayesian Equilibrium profiles do not refine the set of equilibrium outcomes relative to Bayesian Equilibria.

[6]This is a mild assumption since it is not hard to show that Lipschitz continuity of the strategies (and thus differentiability almost everywhere) is a necessary condition for BE. See Ponsatí and Sákovics (1995) for a proof that differentiability is necessary for BE in a very related model.

3. Letting $P^i(t)$ denote the probability that player i concedes at a date $\tau \leq t$, then the boundary condition follows from the requirement that $P^g(0)P^b(0)=0$. (If $P^g(0)>0$ and $P^b(0)>0$ any type conceding at $t=0$ would benefit from a deviation in which she waits to see if the opponent does yield first).

An equilibrium is thus characterized via the ordinary differential equation system obtained from the first order conditions of the players' optimization problems with the appropriate boundary condition. Our next result assures the existence and uniqueness of an equilibrium consistent with observations 1 to 3.

Proposition 3. *The following strategy profile (t^S, t^b) constitutes the unique equilibrium:*

$$t^g(c_g) = \begin{cases} 0 \text{ iff } c_g < g(0), \\ t \text{ iff } c_g = g(t), \\ \infty \text{ iff } c_g \geq G; \end{cases} \quad t^b(c_b) = \begin{cases} 0 \text{ iff } c_b < b(0), \\ t \text{ iff } c_b = b(t), \\ \infty \text{ iff } c_b \geq 1; \end{cases} \tag{6}$$

where g(.) and b(.) are the unique strictly monotone solutions to the system of differential equations

$$g' = \frac{3(C_g - g(t))(1 - b(t))}{2(G - g(t)) - (1 - b(t))}, \quad b' = \frac{(C_b - b(t))(1 + G - g(t) - b(t))}{5(G - g(t)) - (1 + C_b - 2b(t))}, \tag{7}$$

such that

$$b(0) \times g(0) = 0. \tag{8}$$

The detailed Proof of Proposition 3 is in Appendix A. Let us now discuss its implications.

System (7) is the first order condition for types that concede at some $t<\infty$. By type monotonicity, $g'>0$ and $b'>0$. Types that cannot generate surplus never concede, i.e.

$$t^g(c_g) = \infty \text{ for all } c_g > G \text{ and } t^b(c_b) = \infty \text{ for all } c_b > 1. \tag{9}$$

Hence the numerators in Eq. (7) are positive[7], and therefore so must be the denominators, that is $2(G - g(t)) - (1 - b(t)) \geq 0$ and $5(G - g(t)) - (1 + C_b - 2b(t)) \geq 0$. These two inequalities can be rearranged as

$$g(t) \leq \min\left\{ G - \frac{1 + C_b}{5} + \frac{2b(t)}{5}, G - \frac{1 - b(t)}{2} \right\}. \tag{10}$$

Depending on the relative magnitudes of C_b (i.e. the government's uncertainty about the bargaining surplus) and G (i.e. the government's potential concern), one constraint or the other, or both, will be binding. Combining these constraints with the condition on initial values $g(0)b(0)=0$ yield either (i) a solution with $b(0)>0$, where the bargainers have a positive mass of types $c_b<b(0)$ that agree at the outset (i.e. concede to the government);

[7]Recall that $C_g>G$ and $C_b>1$ by assumption.

or (ii) a solution with $g(0)>0$, where the government has a positive mass of types $c_g<g(0)$ that intervene at $t=0$.

Having settled the equilibrium behavior at $t=0$, we remark that as $t\rightarrow\infty$, the unique strictly monotone solution to Eq. (7) compatible with $g(0)b(0)=0$ satisfies

$$\lim_{t\rightarrow\infty} b(t)=1, \quad \lim_{t\rightarrow\infty} g(t)=a\equiv G-\frac{C_b-1}{5}. \tag{11}$$

That is, over time, all bargainers' types that can concede, i.e. with $c_b\leq 1$, eventually do so. For the government, only types $c_g\leq G-\frac{C_b-1}{5}<G$ concede at some $t<\infty$. So even if the government has positive resources to contribute, it might be totally committed not to intervene. This leads naturally to ponder about the probability that the government will intervene in negotiations and contributes part of its stake at any one time t. This is easily measured as the government's type conditional on having positive resources to contribute, that is $\frac{g(t)}{G}$. In the limit as t increases this converges to $\frac{a}{G}=\frac{1}{5}\frac{5G+1-C_b}{G}$, which is bounded away from unity.

These considerations are summarized below:

Proposition 4. *The following hold in the unique equilibrium:*

1. *If G–the maximum stake of the government–is not too large, the bargainers reach a bilateral agreement at t= 0 with (strictly) positive probability.*
2. *In the preceding scenarios, if the government eventually enter a multilateral negotiation, they does so with delay.*
3. *the probability that a government with positive stake, $G-c_g>0$, enter a multilateral negotiation decreases with the range of its uncertainty C_b, and is never greater than $\frac{a}{G}=\frac{1}{5}\frac{5G+1-C_b}{G}$.*

Thus, in the presence of asymmetric information, the unique equilibrium may produce immediate agreement among the bargainers; and it is precisely the bargainers uncertainty about the government's concern that supports immediate agreements—since an unconcerned government will surely let the bargainers to "fight it out" before intervening. In fact, the larger the (maximum) bargainers bilateral surplus is, the less likely intervention is. Provided the government stake is not too high (corresponding to the necessary conditions on G outlined above), there is a positive probability that bargainers will settle their dispute immediately, without government intervention; but the government is still pulled into negotiations with a positive probability that depends on how large the uncertainty on the bargaining surplus is (i.e. how large C_b is). At the other extreme, a very uncertain government with a great potential stake will intervene right away with substantial probability.

4. Conclusion

The upshot of this paper is that the presence of stakeholders in the shadow of bilateral conflicts changes the nature of the negotiations, and inefficiencies are rife: although the model does allow for equilibria where agreement is immediate, delayed equilibria are a pervasive phenomenon. A main prediction of our analysis is thus that industrial relations in sectors of public interest should be characterized by a higher degree of conflict than bilateral negotiations that have no impact over third parties.[8]

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Appendix A

Proof of Proposition 1.

1. *Concession over Wage* (*CW*): The supporting equilibrium strategy profile $s=(t_u, \tau_f, (t_g, \tau_g))$ is $(t_u > t_g^* + t, \tau_f = t, (t_g = t, \tau_g > \tau_f^* + \tau))$, with corresponding equilibrium payoffs $v_u(s) = w_u e^{-r_u t}$, $v_f(s) = h_g e^{-r_f t}$ and $v_g(s) = (S - w_u - h_g)e^{-r_g t}$. To see that this is an equilibrium, consider the union first. Conceding at any later time is payoff irrelevant, as agreement will still be struck at t. It cannot be profitable to concede any earlier either, as this would mean receiving a lower wage at the same time t, since in order to enjoy its payoff the union will still have to wait until agreement is reached in the other set of negotiations at time t. Consider now the firm. By conceding earlier it would obtain the same handout at the same time (since the other set of negotiations is still terminating at t). On the other hand, delaying a concession can only be payoff relevant if the delay exceeds the concession time for the government. Since however $\tau_g > \tau_f^*$ by the definition of τ_f^*, it is suboptimal for the firm to concede at such a later date. Finally consider the government. A deviation to conceding at a later time t' in wage negotiations can have an impact on payoffs only if $t' > t_u > \tau_g^* + t$, in which case the resulting deviation payoff is

$$\left(S - w_g - h_g\right)e^{-r_g t'} > \left(S - w_u - h_g\right)e^{-r_g t} \Leftrightarrow \frac{1}{r_g}\ln\frac{\left(S - w_g - h_g\right)}{\left(S - w_u - h_g\right)} > t' - t > t_g^*. \tag{12}$$

This generates a contradiction, as

$$t_g^* = \frac{1}{r_g}\ln\left(\frac{S - w_g - h_g}{S - w_u - h_f}\right) > \frac{1}{r_g}\ln\left(\frac{S - w_g - h_g}{S - w_u - h_g}\right) \Leftrightarrow h_f > h_g. \tag{13}$$

[8]For instance Gunderson et al., (2001) and Morris (1986) present evidence of this in Canada and the United Kingdom, respectively.

A deviation to some earlier time $t'<t$ in both negotiations[9] would also be non-profitable, as the government's payoff after this deviation would be $(S-w_u-h_f)e^{-r_g t\prime}$, which is smaller than the equilibrium payoff $(S-w_u-h_g)e^{-r_g t}$ since

$$(S-w_u-h_f)e^{-r_g t'}<\left(S-w_u-h_g\right)e^{-r_g t}\Leftrightarrow t-t'<\frac{1}{r_g}\ln\left(\frac{S-w_u-h_g}{S-w_u-h_f}\right), \tag{14}$$

where the last inequality holds true since $t<\overline{t_L}=\frac{1}{r_g}\ln\left(\frac{S-w_u-h_g}{S-w_u-h_f}\right)$.

2. *Concession over Handout* (*CH*): The supporting equilibrium strategy profile $S=(t_u, \tau_f, (t_g, \tau_g))$ is $(t_u=t, \tau_f>t_g^*+t, (t_g>t_u^*+t, \tau_g=t))$, with corresponding equilibrium payoffs $v_u(s)=w_g e^{-r_u}t$, $v_f(s)=h_f e^{-r_f t}$ and $v_g(s)=(S-w_g-h_f)e^{-r_g t}$. Checking that these strategies are a s.p.e. is done in a similar way as the previous case, so we omit details for the sake of brevity. Just observe that a deviation by the government to conceding at a later time τ' in negotiations with the firm is payoff relevant only if $\tau'>\tau_f>t_g^*+t$, in which case the resulting deviation payoff is

$$\left(S-w_g-h_g\right)e^{-r_g\tau'}>\left(S-w_u-h_g\right)e^{-r_g t}\Leftrightarrow\frac{1}{r_g}\ln\frac{\left(S-w_g-h_g\right)}{\left(S-w_u-h_g\right)}>\tau'-t>t_g^*. \tag{15}$$

This is the same as Eq. (12) with τ' in place of t', thus generating a contradiction as before. A deviation to some earlier time $\tau'<t$ in both negotiations[10] would also be non-profitable, as the government's payoff after this deviation would be $(S-w_u-h_f)e^{-r_g\tau'}$ which is smaller than the equilibrium payoff $(S-w_g-h_f)e^{-r_g t}$ since

$$(S-w_u-h_f)e^{-r_g\tau'}<\left(S-w_g-h_f\right)e^{-r_g t}\Leftrightarrow t-\tau'<\frac{1}{r_g}\ln\left(\frac{S-w_g-h_f}{S-w_u-h_f}\right), \tag{16}$$

where the last inequality holds true since $t<\bar{t}_R=\frac{1}{r_g}\ln\left(\frac{S-w_g-h_f}{S-w_u-h_f}\right)$.

3. *No Concessions* (*NC*): The supporting equilibrium strategy profile $S=(t_u, \tau_f, (t_g, \tau_g))$ is $(t_u>t, \tau_f=t, (t_g>t_u^*+t, \tau_g>\tau_f^*+\tau))$, with corresponding equilibrium payoffs $v_u(s)=w_g e^{-r_u t}$, $v_f(s)=h_g e^{-r_f t}$ and $v_g(s)=(S-w_g-h_g)e^{-r_g t}$. Again, we omit a complete verification of the optimality of these strategies, which is analogous to what outlined in the cases above. Just note that the only potentially profitable deviation for the government is to concede before t in both negotiations—if the government conceded in only one bargain, it would decrease its payoff in that negotiation without any impact on the implementation date. Thus in a deviation the government would set $t_g=\tau_g=t-\varepsilon\geq 0$. The corresponding payoff is thus $S-w_u-h_f$, so that this deviation is not profitable since

$$(S-w_u-h_f)e^{-r_f(t-\varepsilon)}<\left(S-w_g-h_g\right)e^{-r_g t}\Leftrightarrow\varepsilon<\frac{1}{r_f}\ln\left(\frac{S-w_g-h_g}{S-w_u-h_f}\right)=\bar{t}_I, \tag{17}$$

which concludes the proof. □

[9]Recall that if the government were to give in earlier in wage negotiations over wages only, the overall time of agreement would be left unchanged, but a higher wage would be settled.

[10]Recall that if the government were to give in earlier in wage negotiations over handout only, the overall time of agreement would be left unchanged, but a higher handout would be settled.

Proof of Proposition 2. To avoid unnecessary complication we abuse notation here and define t_g^*, t_u^*, τ_f^* and τ_g^* as in Section 2 as the point in time when agent i is indifferent between conceding immediately to the opponent and obtaining the most preferred outcome, where now preferred outcomes correspond to the bounds $\underline{w}$, $\bar{w}$, $\underline{h}$ and $\bar{h}$. Strategies supporting the equilibria in Proposition 2 are as follows:

1. '*Leftwing government*' *equilibrium* (*LGE*):

$$\tilde{s} = \left(\left(w_u = \overline{w}, t_u > t_g^* + \tau\right), (h_f = h, \tau_f = \tau), \left(w_g = w, h_g = \underline{h}, t_g = \tau, \tau_g > \tau_f^* + \tau\right)\right). \tag{18}$$

2. '*Rightwing government*' *equilibrium (RGE):*

$$\tilde{s} = \left((w_u = w, t_u = \tau), \left(h_f = \overline{h}, \tau_f > \tau_g^* + \tau\right), \left(w_g = \underline{w}, h_g = h, t_g > t_u^* + \tau, \tau_g = \tau\right)\right). \tag{19}$$

3. '*Intransigent government*' *equilibrium (IGE):*

$$\tilde{s} = \left((w_u = w, t_u = \tau), (h_f = h, \tau_f = \tau), \left(w_g = \underline{w}, h_g = \underline{h}, t_g > t_u^* + \tau, \tau_g > \tau_f^* + \tau\right)\right). \tag{20}$$

Consider the war of attrition phase first. Substitution of $w_u = \bar{w}$, $w_g = w$, $h_f = h$ and $h_g = \underline{h}$ into $\bar{t}_L$ from the Proof of Proposition 1 yields $\tau_L = \bar{t}_L$, so that by Proposition 1 those exhibited are strategies supporting the LGE in the war of attrition stage. Similarly, for the RGE after letting $w_u = w$, $w_g = \underline{w}$, $h_f = \bar{h}$ and $h_g = h$ into $\bar{t}_R$, so that $\bar{\tau}_R = \bar{t}_R$; and for the IGE after substituting $w_u = w$, $w_g = \underline{w}$, $h_f = h$ and $h_g = \bar{h}$, so that $\bar{\tau}_I = \bar{t}_I$. Consider now deviations in the first stage, starting with the union. If it were to deviate to a compatible demand, i.e. to some $w' \leq \bar{w}$, its payoff would not be affected, as because of incompatible demands in the parallel set of negotiations the game would still enter the second stage, with termination at time $t = \tau$. Similarly for the firm. Consider now the government. For equilibrium LGE in case 1, a deviation in the first stage to compatible payoffs would yield the government

$$S - \overline{w} - h < \left(S - \overline{w} - \underline{h}\right) e^{-r_g \tau} \Leftrightarrow \tau < \frac{1}{r_g} \ln\left(\frac{S - \overline{w} - \underline{h}}{S - \overline{w} - h}\right) = \bar{\tau}_L. \tag{21}$$

Consider now the equilibrium RGE of case 2 above. A deviation in the first stage to compatible payoffs would yield the government

$$S - w - \overline{h} < \left(S - \underline{w} - \overline{h}\right) e^{-r_g \tau} \Leftrightarrow \tau < \frac{1}{r_g} \ln\left(\frac{S - \underline{w} - \overline{h}}{S - w - \overline{h}}\right) = \bar{\tau}_R. \tag{22}$$

Finally consider equilibrium IGE. The government's equilibrium continuation payoff in the second stage is $(S-\underline{w}-\bar{h})\mathrm{e}^{-r_{\mathrm{g}}\tau}$. By deviating to compatible demands[11] the government would get $S-w-h$ immediately. But

$$S-w-h<\left(S-\underline{w}-\underline{h}\right)\mathrm{e}^{-r_{\mathrm{g}}\tau}\Leftrightarrow\tau<\frac{1}{r_{\mathrm{g}}}\ln\left(\frac{S-\underline{w}-\underline{h}}{S-w-h}\right)=\overline{\tau}_{\mathrm{I}}. \tag{23}$$

To conclude, observe that in the bargaining phase for the player who anticipates that his proposal will prevail in the second stage, it is suboptimal to set his claim/offer to a value which is different from either the highest (for either the union or the firm) or the lowest (for the government). Thus those characterized above are the only equilibrium configurations possible for delayed agreement in pure strategies. □

Proof of Proposition 3. Fix an equilibrium profile $(t^{\mathrm{b}}, t^{\mathrm{g}})$. That is, $t^{\mathrm{b}}(c_{\mathrm{b}})$ maximizes $\int_0^t \frac{G+1-g(\tau)-c_{\mathrm{b}}}{3}\mathrm{e}^{-\tau}\mathrm{d}P^{\mathrm{g}}(\tau)+(1-P^{\mathrm{g}}(t))\frac{(1-c_{\mathrm{b}})}{2}\mathrm{e}^{-t}$ and $t^{\mathrm{g}}(\boldsymbol{c}_{\mathrm{g}})$ maximizes $\int_0^t\left(G-c_{\mathrm{g}}\right)\mathrm{e}^{-\tau}\mathrm{d}P^{\mathrm{b}}(\tau)$ $+(1-P_{\mathrm{b}}(t))\frac{G+1-c_{\mathrm{g}}-b_t\left(c_{\mathrm{g}}\right)}{3}\mathrm{e}^{-t}$, where $b_t(c_{\mathrm{g}})=E(c_{\mathrm{b}}|b(t)<c_{\mathrm{b}}\leq G+1-c_{\mathrm{g}})$.

The following claims are important, their proofs are standard and omitted for the sake of brevity.

1. Equilibrium strategies are type monotone.
2. $t^{\mathrm{g}}(c_{\mathrm{g}})=\infty$ for all $c_{\mathrm{g}}>G$ and $t^{\mathrm{b}}(c_{\mathrm{b}})=\infty$ for all $c_{\mathrm{b}}>1$.
3. $P^{\mathrm{g}}(0)P^{\mathrm{b}}(0)=0$.

Claims 2 and 3 imply that there are types such that $t^i(c_i)=t\in(0,\infty)$, and the necessary first order condition for this types implies that the inverse of $(t^{\mathrm{b}}, t^{\mathrm{g}})$ must be characterized by a solution to the autonomous dynamical system (7).

By Claim 1, the relevant solution of Eq. (7) must be strictly increasing; and by Claim 3 its initial condition must be in the set $I=\left\{(x,y)\in\Re^2 \text{ such that } x,y=0\right\}$. Consider the open set D,

$$D=\left\{(x,y)\in(-\epsilon,1)\times(-\epsilon,G),\ \text{such that } y<\min\left\{G-\frac{1}{2}+\frac{x}{2},G-\frac{1-C_{\mathrm{b}}}{5}+\frac{2x}{5}\right\}\right\}. \tag{24}$$

Note that a solution of Eq. (7) such that $(g(t), b(t))\notin D$ for some, $0<t<\infty$, cannot describe an equilibrium strategy profile: either because it is decreasing, or because it prescribes that types $c_{\mathrm{g}}>G$ or $c_{\mathrm{b}}>1$ choose a finite termination date. By the Fundamental Theorem of ordinary differential equations, a unique solution to Eq. (7) goes though each $(x,y)\in I\cap D$. And observe, moreover, that each such solution is strictly increasing, and approaches the boundary of D. Consider the point $(1, a)$, $a=G-\frac{C_{\mathrm{b}}-1}{5}$ in the boundary of D, and note that there is a unique $(x,y)\in I\cap D$ such that the solution to Eq. (7) through (x, y) approaches $(1, a)$. Denote this unique point (γ^*, β^*) and let (g^*, b^*) denote the unique solution to Eq. (7) with initial condition (γ^*, β^*). Observe that for all t, $0<t<\infty$,

[11]This deviation must affect both demand games, as otherwise the war of attrition stage would be triggered, and the government would still receive their payoff with delay, but after having contributed larger amounts.

$b^*(t) < 1$ and $g^*(t) < a$, and $\lim_{t\to\infty} b^*(t) = 1, \quad \lim_{t\to\infty} g^*(t) = a$. To see that condition (11) is indeed necessary, assume that the equilibrium strategy profile is described by another solution to Eq. (7) $(\tilde{g}, \tilde{b})$, with initial condition $(\gamma, \beta) \in I \cap D$, $(\gamma, \beta) \neq (\gamma^*, \beta^*)$. Since $(\tilde{g}(t), \tilde{b}(t))$ approaches the boundary of D at $(z, u) \neq (1, a)$, then either (i) $z = 1$, $u < G$; or (ii) $z < 1$, $u = G - \frac{1+C_b}{5} + \frac{2z}{5}$; or (iii) $z < 1$, $u = G - \frac{1}{2} + \frac{z}{2}$. Consider case (i): then $(\tilde{g}(T), \tilde{b}(T)) = (z, u)$ for $T < \infty$, that is, no type of either player terminates the game after t; this cannot be equilibrium behavior since any type c_g, $g(T) < c_g < G$, such that $t^g(c_g) = \infty$ according to the alleged strategy, is strictly better off deviating to $t'^g(c_g) = T + \Delta$. In cases (ii) and (iii) $t^b(c_b) = \infty$ for $c_b < 1$. Along this profile, for each $\pi > 0$, there is a $t_\pi < \infty$, such that $P(t^g(c_g) < \infty | t^g(c_g) \geq t_\pi) \leq \pi$; and for each $c_b < 1$ there is a $\pi_b > 0$ such that if $P(t^g(c_g) < \infty | t^g(c_g) \geq t_\pi) \leq \pi_b$, then $\int_t^\infty \frac{G+1-g(\tau)-c_b}{3} e^{-\tau} dP^s(\tau) \leq \pi_b \frac{G+1-g(\tau)-c_b}{3} e^{-t} < \frac{(1-c_b)}{2} e^{-t}$, contradicting that $t^b(c_b) = \infty$ is a best response for any $c_b < 1$. Hence the equilibrium profile must be characterized by (g^*, b^*). Checking that the single crossing property implies that necessary conditions are also sufficient for an equilibrium completes the proof. □

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