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The deadline effect: A theoretical note☆

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Abstract

A two-person concession game with a deadline is presented. It is proved that along the Bayesian equilibrium players tend to reach an agreement at the deadline.

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1. Introduction

Last minute agreements are a pervasive phenomenon in real-life negotiations as well as in experimental ones (see Roth et al., 1988). This paper presents a simple game of concession in which the combination of a deadline with two-sided incomplete information leads to a unique Bayesian equilibrium (BE) with a salient deadline effect.

An effective deadline yields an exogenous discontinuity in the payoffs that agents can enjoy over time. A natural deadline in labor–management negotiations may be the date at which the contract expires. Other examples are the date at which the firm expects to run out of inventories or the trial date in pre-trial negotiations.

Ponsati and Sakovics (1995a) characterize BE for concession games with incomplete information without a deadline. Here we consider the effect of introducing a deadline: along the unique BE the distribution of dates of agreement is continuous in $(0, T)$ and has mass points at T and possibly at $t = 0$. Moreover, there is some date $\underline{t}$, $0 < \underline{t} < T$, such that the

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probability of concession is nil in the interval $(\underline{t}, T)$. Thus, our model exhibits a strong deadline effect.

Among the continuum of Nash equilibria of complete information, continuous time, wars of attrition characterized by Hendricks et al. (1988), there are mixed strategy equilibria similar to the one we exhibit: a mass point of concessions at the deadline and no concessions on some interval preceding it. Ma and Manove (1993) study a complete information bargaining game in which agents face a deadline. Offers and counteroffers are exchanged with random delays so that, as the game approaches the deadline, the chances of 'accidentally' missing the deadline increase. They show that along the only equilibrium that is symmetric and Markov-perfect, early offers are made and rejected, agreements are reached late in the game and the deadline is missed with positive probability.

Since games of the war of attrition are a rather crude approach to bargaining problems, it would be pretentious to claim that our model says much about deadlines in bargaining. However, a concession game can, in some cases, be taken as a reduced form of a richer game (see, for example, Ponsati, 1990, and Ponsati and Sakovics, 1995b). In this context, our results suggest that in very polarized negotiations a credible deadline has a very positive effect. In this case, since conceding means giving up almost all the surplus, most types do not concede, and the probability that the opponent concedes is arbitrarily small. Thus, the average gains from trade without a deadline are arbitrarily close to zero. An early deadline would yield a strictly positive probability of a compromise at the deadline, which clearly yields positive average gains.

2. The model

Consider two agents disputing over two alternatives A and B. Player a favors A, player b favors B. Let us denote a generic alternative by X (similarly, denote a generic player by x). Then player i's $(i = a, b)$ static preferences can be described as follows:[1]

$$u_i(X, s) = \begin{cases} 1 - s & \text{if } i = x , \\ -s & \text{if } i \neq x , \end{cases}$$

where s, his type, is a privately known parameter (reservation value).

The game is played in continuous time, starting at $t = 0$, and ending at the deadline T, $T < \infty$. The players have a unique action available to them – though they can freely choose its timing – they can *yield*. Player a (b) starts out proposing alternative A (B) and this situation persists until she yields, at which time B (A) is implemented. In case both players concede at the same time, they use a lottery that selects the preferred outcome of player i with probability π^i, $0 < \pi^i < 1$. Thus, a strategy for player i, σ_i is a (measurable) function from his type to the (possibly infinite) time of his concession.

[1] We use the above preferences for simplicity. We actually only need the following restrictions on preferences (assume we deal with Player a): $u(B, s) \geqslant u(\text{Disagreement})$ iff s is non-positive; $u(A, s) > u(B, s)$ for all s; $\partial u(X, s)/\partial s < 0$; and $u(B, s') - u(B, s) \geqslant u(A, s') - u(A, s)$ for $s' \geqslant s$.

Player i receives (intertemporal) utility $U_i(X, s, t) = u_i(X, s)e^{-t}$ if alternative X is reached at time t.[2] If no player yields at $t \leq T$, both players receive a zero payoff.

We assume that a priori beliefs on players' types are common knowledge and they can be represented by a probability distribution F_i, with positive density f_i, over the interval $[i_{\mathrm{L}}, i_{\mathrm{H}}]$, $f_i \in C^{\infty}$. We assume that $i_{\mathrm{L}} < 0 < i_{\mathrm{H}} \leq 1$. That is, with positive probability both players are of a type who derives a positive utility even if the opponent's alternative prevails (weak type); with positive probability both players are of a type who derives a negative utility if the opponent's alternative prevails (strong type); and there are no types of either player who derive negative utility even if their preferred alternative is agreed upon. Finally, we assume that the type of player a is independent from player b's type.

Given a strategy profile σ, let $(X(\sigma(s, z)), t(\sigma(s, z)))$ be the outcome generated by σ if types are s and z, and let $V_i^s(\sigma)$ denote the expected payoff to player i of type s given σ:

$$V_i^s(\sigma) = \int_{[j_{\mathrm{L}}, j_{\mathrm{H}}]} U_i(X(\sigma(s, z)), s, t(\sigma(s, z)))\, \mathrm{d}F_j(z) .$$

Then σ is a *Bayesian equilibrium* (BE) if and only if for all (s, z) and $i = a, b$: $V_i^s(\sigma) \geq V_i^s(\sigma_i', \sigma_{-i})$ for all σ_i'.

3. The result

Ponsati and Sakovics (1995a) characterize Bayesian equilibrium outcomes when $T = \infty$. Here we address the case $T < \infty$.

Proposition. Let $\varepsilon: (0, \underline{t}) \to (i_{\mathrm{L}}, 0)$ *and* $\zeta: (0, \underline{t}) \to (j_{\mathrm{L}}, 0)$ *be increasing, differentiable functions that uniquely solve the following system of differential equations:*

$$\begin{aligned} -\zeta'(t) f_j(\zeta(t)) &= [1 - F_j(\zeta(t))]\varepsilon(t) , \\ -\varepsilon'(t) f_i(\varepsilon(t)) &= [1 - F_i(\varepsilon(t))]\zeta(t) , \end{aligned} \tag{1}$$

with $\lim_{t \to \underline{t}} (\varepsilon(t), \zeta(t)) = (\underline{\varepsilon}, \underline{\zeta})$ *and* $(\lim_{t \to 0} \varepsilon(t) - i_{\mathrm{L}})(\lim_{t \to 0} \zeta(t) - j_{\mathrm{L}}) = 0$.
And let $(\underline{\varepsilon}, \underline{\underline{\varepsilon}}, \underline{\zeta}, \underline{\underline{\zeta}})$ *and* $\underline{t}$ *solve*

$$\begin{aligned} -\underline{\varepsilon}\, \mathrm{e}^{-\underline{t}} &= \left\{ \frac{F^j(\underline{\zeta}) - F^j(\underline{\underline{\zeta}})}{1 - F^j(\underline{\zeta})} (\pi^i - \underline{\varepsilon}) - \frac{1 - F^j(\underline{\underline{\zeta}})}{1 - F^j(\underline{\zeta})} \underline{\varepsilon} \right\} \mathrm{e}^{-T} , \\ \frac{F^j(\underline{\underline{\zeta}}) - F^j(\underline{\zeta})}{1 - F^j(\underline{\zeta})} (\pi^i - \underline{\underline{\varepsilon}}) - \frac{1 - F^j(\underline{\underline{\zeta}})}{1 - F^j(\underline{\zeta})} \underline{\underline{\varepsilon}} &= \frac{F^j(\underline{\underline{\zeta}}) - F^j(\underline{\zeta})}{1 - F^j(\underline{\zeta})} (1 - \underline{\underline{\varepsilon}}) . \end{aligned} \tag{2}$$

Then, the BE strategies are

[2] Again, the assumption on time preferences is for simplicity. It would suffice to have $U_i(X, s, t) = u_i(X, s)\phi(t)$, with ϕ continuously differentiable, strictly decreasing and $\lim_{t \to \infty} \phi(t) = 0$.

(i) *for* $s < \underline{\varepsilon}$, $\sigma_i(s) = t$ *if and only if* $\varepsilon(t) = s$ (*for* $z < \underline{\zeta}$, $\sigma_j(z) = t$ *if and only if* $\zeta(t) = z$),
(ii) *for* $\underline{\varepsilon} \leq s < \underline{\underline{\varepsilon}}$, $\sigma_i(s) = T$ (*for* $\underline{\zeta} \leq z < \underline{\underline{\zeta}}$, $\sigma_j(z) = T$),
(iii) *for* $\underline{\underline{\varepsilon}} \leq s$, $\sigma_i(s) = \infty$ (*for* $\underline{\underline{\zeta}} \leq z$, $\sigma_j(z) = \infty$).

Proof. Let $H_i^{\sigma}(t)$ denote the probability that i gives in no later than t according to σ. We will write $H_i(t)$ if no confusion arises. Correspondingly, $H_i(\infty)$ will denote the probability that i will ever concede. Moreover, let $\Sigma_i(t) = H_i(t) - \lim_{\delta \to 0} H_i(t - \delta)$; that is, $\Sigma_i(t)$ is the probability that i gives in exactly at t.

(Necessity) *Step 1. Eq.* (*1*) *must hold.* By Lemmas 1 to 4 and 6 to 9 in Ponsati and Sakovics (1995a) the supports of the distribution of concession times are the same for both players, $[0, t]$, have no gaps ending before $\underline{t}$ and are differentiable in $(0, \underline{t})$. Hence, the first-order condition (1) must hold in $(0, \underline{t})$. At the same time, by Lemma 5 in Ponsati and Sakovics (1995), we need initial condition $(\lim_{t \to 0} \varepsilon(t) - i_{\mathrm{L}})(\lim_{t \to 0} \zeta(t) - j_{\mathrm{L}}) = 0$.

Step 2. Both players must have a mass point at T. Assume that i's strategy has a mass point at T while j's does not. There must be some $\mu > 0$ such that no type of j makes any concession in $(T - \mu, T)$. If this is the case, however, any type of i conceding at T will increase payoff by conceding a bit earlier at some t in $(T - \mu, T)$.

Assume that neither player concedes at T with positive probability. If there are $s < 0$ that do not concede at $t < T$, since we are assuming that the opponent concedes at T with zero probability, then they all prefer to deviate and concede at T. Thus, all $s \leq 0$ concede at $t \leq T$. We will first consider the case $\underline{t} < T$. Consider s such that $\sigma_i(s) = t < \underline{t}$, he gets a payoff $\int_{[0,t)} (1 - s)\, \mathrm{e}^{-\tau}\, \mathrm{d}H^j(\tau) - (1 - H^j(t))s\, \mathrm{e}^{-t}$, while he gets $\int_{[0,T')} (1 - s)\, \mathrm{e}^{-\tau}\, \mathrm{d}H^j(\tau) + \int_{(t,T')} (1 - s)\, \mathrm{e}^{-\tau}\, \mathrm{d}H^j(\tau) - (1 - H^j(T'))s\, \mathrm{e}^{-t'}$, if he deviates and concedes at t' in $(t, T]$. Since $H^j(\cdot)$ is strictly increasing at any interior $t < \underline{t}$, for s close enough to 0 and t' close enough to $\underline{t}$, $(H^j(t') - H^j(t))\, \mathrm{e}^{-t'} - (1 - H^j(t))s(\mathrm{e}^{-t'} - \mathrm{e}^{-t}) > 0$, i.e. conceding at t' is a profitable deviation. Hence, if there are no mass points of concessions at T, then all $s \leq 0$ concede at $t < \underline{t} = T$. Therefore, $\sigma_i(T) = 0$ are terminal conditions for Eq. (1), but the solution to (1) that such a terminal condition yields will be incompatible with $(\lim_{t \to 0} \varepsilon(t) - i_{\mathrm{L}})(\lim_{t \to 0} \zeta(t) - j_{\mathrm{L}}) = 0$. Consequently, both players have a positive mass of types that concede at T.

Step 3. Last date of concession before T. Since $\pi^i < 1$, a mass point at T implies that $\underline{t} < T$, i.e. no type of either player concedes in $(\underline{t}, T)$.

Step 4: Eqs. (2). The first expression in (2) is necessary because $\underline{\varepsilon}$ and $\underline{\zeta}$ must be indifferent between concession at T or $\underline{t}$; the second is necessary because $\underline{\underline{\varepsilon}}$ and $\underline{\underline{\zeta}}$ must be indifferent between concession at T or no concession ever. Finally, $\lim_{t \to \underline{t}} (\varepsilon(t), \zeta(t)) = (\underline{\varepsilon}, \underline{\zeta})$ is also necessary because the $\underline{\varepsilon}$, $\underline{\zeta}$, that we obtain from (2) as a function of $\underline{t}$, must be terminal conditions to (1).

(Sufficiency) It is immediate to check that neither a player i of type $\varepsilon(t)$ nor players conceding at T or not conceding can increase their payoffs by changing their behavior. Hence the proposed conditions are also sufficient for a BE. □

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