The continuation of bargaining by other means: an elementary theory of war

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Abstract

We propose an elementary theory of wars fought by fully rational contenders. Two parties play a Markov game that combines stages of bargaining with stages that are a contest to attain or resist *advantage* - the ability of one side to impose surrender on the other. Under uncertainty and incomplete information, in the unique equilibrium, long confrontations occur: war arises when reality disappoints initial (rational) optimism, and it persist longer when both agents are optimists but reality proves both wrong. Bargaining proposals that are rejected initially might eventually be accepted after several periods of confrontation. We provide an explicit computation of the equilibrium, evaluating the probability and durability of war, and its expected losses as a function of i) the costs of confrontation, ii) the asymmetry of the split imposed under surrender, and iii) parameters messuring the strengths of the agents at attack and defense. The effects of changes in the environment are non-monotone.

keywords: bargaining, commitment, war.

1 Introduction

We propose an elementary theory of wars fought by fully rational contenders. Consider two parties that must resolve a dispute that could potentially go on for ever. This interaction is modeled as a multi-stage Markov game, where over time, players may find themselves in a *bargaining state*, where one party makes a proposal and its acceptance by the other ends the game, or they may be in a state where bargaining is suspended, a *claim state*. In the later states, one side is committed to a special outcome biassed to her benefit, her claim, and the game can terminate only with the opponent's surrender to such claim. Upon rejection of a proposal or when a claim mets opposition, the game continues with the conflict unresolved, which is costly to both parties. In the following period, the state of the game can change from a bargaining state into a claim state, or vice-versa. This changes are randomly determined by transition probabilities which depend on the strength of each side. For example, the two parties aim to control some territory that contains a landmark of great value (an oleoduct, an emblematic city, or a diamond mine). Assume that temporary control of the landmark is obtained or lost, with randomness, through confrontation. Then claim states are those where one of the parties is (temporary) in control of the landmark. The game can terminate when one side claims control of the landmark and the opponent surrenders, or else, in a bargaining state, by an agreement to share the territory and the landmark.

Willingly withdrawing a claim earned by force quickly undermines an army's reputation, and this is very costly in future or concurrent conflicts.[1] Consequently, a crucial assumption of our analysis is that claims entail a commitment - i.e. that a party attaining (temporary) control of the landmark is not free to back up and bar-

[1]The notion of honor is related. Armies are trained to defend honor, that is, to sustain commitment to claims - even when they are hopeless - regardless of the cost, and to maintain a reputation for doing so. See O'Neil (1999).

gain an agreements where she losses (some of) its control, until she is defeated in confrontation.[2]

Our analysis takes off with the characterization of equilibria under complete information, when the size of potential claims (the value of the landmark) and the strength of the players are known. Regardless of the parameters, in bargaining states, agreement prevails immediately in equilibrium. As usual in alternating-offer bargaining, the proposer offers a share that leaves the responder just indifferent to her continuation value upon confrontation, which is accepted. The (potential) gains attainable in claim states may determine the terms of agreement, but players never resort to confrontation. Exploring the circumstances in which claims are effective and induce surrender provides the main insight in this part of the analysis: When a claim is established, either it induces surrender - so that it immediately awards permanent control over the landmark to its temporary claimant - or else it is met by opposition until it is dismissed by force. Very extreme claims (relative to their persistence upon confrontation) give such a small payoff to the opponent that she will never surrender. (This is a simple but fundamental and ancient principle; Sun Tzu's Art of War of 510 BC, advises: "Do not press a desperate foe too hard" (Sun Tzu (1988)). Thus, when claims are sufficiently great, confrontation prevails at every period until the bargaining state is re-established. This observation becomes crucial when we address the effect of uncertainty and private information.

In the second part of the paper we drop the assumption that transitions between states are governed by probabilities are constant and well known, and we introduce asymmetric information. We display environments where 'War is a dispute about the

[2]Frictions in decision making, which are common phenomenon when each side is a group of heterogeneous individuals, are also relevant. Groups often raise claims thanks to the efforts of their more radical members, who are then likely to control decisions at least for a while: 'while it is hard for a government, particularly a responsible government, to appear as irrational whenever such appearance is expedient, it is equally hard for a government, particularly a responsible one, to *guarantee* its own moderation in every circumstance.'(Schelling (1966), p. 41).

measurement of power' (Blainey (1988), p. 114.), which may arise, eventhough the game starts in a bargaining state: In the unique equilibrium strategy profile a very rich range of outcomes may occur. For natural parameter configurations war starts with positive probability. This confrontation may lead to a claim state, or not. If the claim state does not induce immediate surrender because the claim is too large, a (potentially very long) phase of confrontation follows until the claim is dismissed. In bargaining sates, proposals that initially are rejected to engage in war may eventually be accepted after a long war.

The intuition for these results is the following. The power to sustain claims is always uncertain, but each party is privately informed about (the probability distribution of) her own power. Hostile (lenient) types have a high (low) expectation of claim persistence; and consequently expect high (low) payoffs in claim states. In bargaining states -before any claim is established - the proposer can extract a large share from a lenient responder because such type expects low returns from confrontation. Suppose that the prior probability that the opponent is a lenient type is high so that the proposer's optimism - her anticipation that she can get a large share - has rational basis. Then, in equilibrium, the proposer demands the large share which is acceptable only by the a lenient type. But when the responder turns out to be a hostile type, rejection follows and so war starts. If a claim is established in the sequel, and its realized persistence is insufficient to induce surrender, war will continue.

As we obtain a unique equilibrium, which is easily computed, our model allows precise comparative statics: We can measure the effect of changes of the different parameters on the likelihood, duration, and costs of war. We find that war occurs with positive probability provided that agents' belief that the opponent is a lenient type is greater that a given threshold. As a function of confrontation cost, claims' size, and agents' strength, this threshold is increasing. Thus, departing from situations where war occurs with positive probability, small parameter changes are irrelevant; but a sufficient increase drops the probability of war to zero. A similar non-monotonicity holds with respect to duration and losses.

We carry out our analysis for the simplest scenario, assuming that there are only two states, and that transitions are governed by stationary probabilities. Real conflicts have immense sets of states and their transition probabilities are non stationary. Nevertheless, as the qualitative nature of our results does not rely on this drastic simplifications, our intuitions are relevant for the analysis of real more complex disputes.

The rest of the paper is organized as follows. Section 2 relates our work to the literature. The model is formally presented in Section 3. Section 4 discusses environments of complete information. The main results are in Section 5 where incomplete information is assumed.

2 Relation to the literature

That war arises in strategic interactions where bargaining takes place along with a contest to attain advantage was advocated by Clausewitz in his classical treaty of 1832, which has inspired many others. Among these Blainey (1988) is specially insightful. He argues: "unfair advantages are a characteristic of war. In every war, it seems, at least one of the nations agrees to fight because it believes that it is stronger than the enemy, because it believes that it possesses an unfair advantage (...). Likewise in each period of peace larger nations peacefully exercise power in preserving their own interests simply because they possess that unfair advantage" (Blainey (1988) p. 169). In the strategic literature, it has long been recognized that unilateral commitment awards advantage; and that attempts to attain commitment or to dismiss that of opponents' are a fundamental source of conflict (Crawford (1982), Schelling (1960)).

The fundamental role of asymmetric information in generating inefficient bargaining outcomes, i.e. fueling costly conflict, is well known in the theoretic literature. Banks (1990), Bester and Warneryd (1998), Fearon (1995) and Powell (1996), (1999) propose game-theoretic analysis of war focusing on the role of asymmetric information in prompting disagreement in negotiations *prior* to fighting - taking war as an outside option. As war is considered a game-ending move, the scope of this models is limited to the analysis on the origins of war.

Wagner (2000) argues that because wars are the processes by which parties learn each others real forces and costs - thus opening the door to agreements that are impossible without war - the focus of analysis ought to be on bargaining while fighting. Although he does not offer formal results, his discussion of the process by which wars start, develop and end is suggestive: wars commence because inconsistent expectations on the consequences of fighting initially prevent the existence of agreements that both parties prefer to confrontation; as fighting proceeds expectations are adjusted and mutual gains from agreement arise. Filson and Werner (2002), Smith and Stam (2004), Slanchev (2003), and Powell (2004) present formal explorations of these intuitions. Filson and Werner (2002) discuss a very special (two-period, two-type) formulation, and emphasize the role of battlefield resource availability. Smith and Stam (2004) and Slanchev (2003) build on Smith (1998) model war as a Markov game taking place along with bargaining, and focus on the effect of uncertainties on military power. Powell (2004) models war as a costly process of bargaining during which parties run the risk of military collapse; he considers uncertainties over either power or the cost of fighting, which allows comparisons of the learning processes induced in each case. Filson and Werner (2002), Slanchev (2003) and Powell (2004) assume one-sided private information with common priors, show that separating equilibria can be sustained, and provide insights on the process of information revelation that unravels through war based on the properties of equilibria. However, as Powell (2004) emphasizes, these results are extremely sensitive to the details of the game or the equilibrium notion. A general problem is all these models is the great multiplicity of equilibria, which makes comparative statics very problematic. Furthermore, the assumption that one of the parties is fully informed has such powerful implications that conclusions cannot extend to setups where the agents strategic capabilities are relatively symmetric. Smith and Stam (2004) argue that war arises due to disagreements on beliefs, and formulate such disagreement as different a priori beliefs on military capabilities. Their approach is effective to supply tractable and transparent predictions. Such predictions, however, rely on the specific details of these beliefs; this requires *common knowledge of the non-common priors*, a problematic assumption, which inconsistent with any presumption of full rationality by the parties.

The present paper shares Smith and Stam (2004) premise that disagreements on beliefs are crucial, but maintains the analysis in the realm of Bayesian games with common priors. In contrast with the preceding literature, our treatment of informational issues is perhaps too simple for a rich theory of war as a process of information transmission - in our model private information triggers war, but it is commitment that sustains it in the continuation. However, thanks to the simplicity of the informational set up, our model delivers a unique equilibrium - featuring long confrontations and fortune reversals - which is straightforward to characterize, and allows a symmetric treatment of the agents.

3 The game

Two players $i = 1, 2$ must split one unit of surplus. Time is discrete $t = 0, 1, 2, ...$ The state of the game establishes the moves available for each player at each date. There are four possible states, two bargaining states s_{bi}, where no player holds a claim and bargaining proceeds with an exchange of proposals, and two claim states, s_{ci} in which one player holds a claim and bargaining is suspended. In state s_{bi}, player i is the proposer, she chooses a proposal, any pair $(x_i, 1 - x_i)$, $0 \leq x_i \leq 1$, then, player j, decides whether to accept or reject. On the other hand, in state s_{ci}, only player $j \neq i$ moves, and her choices are opposition or surrender. Surrender terminates the game, i takes her claim c_i, $1/2 < c_i \leq 1$, and j gets $1 - c_i$. Upon rejection or opposition, one period of confrontation takes place and the game moves to the following period. When confrontation occurs at t the state at $t + 1$, s_{t+1}, is randomly determined with probabilities that depend on the state at t, s_t, according to the transition probability matrix of Table 1.

$t \backslash t+1$	s_{b1}	s_{b2}	s_{c1}	s_{c2}
s_{b1}	$\frac{p_{2t}}{2}$	$\frac{p_{2t}}{2}$	0	$1 - p_{2t}$
s_{b2}	$\frac{p_{1t}}{2}$	$\frac{p_{1t}}{2}$	$1 - p_{1t}$	0
s_{c1}	$\frac{1-q_{1t}}{2}$	$\frac{1-q_{1t}}{2}$	q_{1t}	0
s_{c2}	$\frac{1-q_{2t}}{2}$	$\frac{1-q_{2t}}{2}$	0	q_{2t}

Table 1: The matrix of transition probabilities between states.

The parameters q_{it} and $1 - p_{it}$ measure i's strength, respectively at defense and attack: q_{it} is the probability that player i will maintain her claim into period $t + 1$ (i.e. the persistence of player i's claim at t) while $1 - p_{it}$ is the probability that player i establishes a claim when she rejects a proposal in bargaining state s_{bj}.

Confrontation is costly. When the outcome of the game is perpetual confrontation both agents obtain 0. Upon a termination that allocates shares $(z_i, 1 - z_i)$ at date t agent i obtains $\delta^t z_i$, with $0 < \delta < 1$.

A *history of the game* at t is a sequence of states of the game from 0 to t, the rejected proposals or opposed claims from 0 to $t-1$, and possibly a standing proposal. A *strategy* for player i, denoted σ_i, selects the action of player i at each history in which she must move. A strategy profile is a subgame perfect equilibrium if, at every history, the actions of both players are mutually best responses. A strategy profile is *stationary* if actions depend only on the state of the game. We use the term *equilibrium* to refer to a subgame perfect equilibrium in stationary strategies.

Our results are robust to richer formulations of the extensive form. Results remain unchanged when the responder chooses between acceptance, rejection without confrontation, and rejection with confrontation. At the cost of increased complexity we can consider larger state sets - for example a state in which both players hold incompatible claims; or we can admit richer transitions probabilities - for example c_j could follow directly from c_i. The cost of confrontation could also be asymmetric across players and or states.

The present game is related to the general class of bargaining games studied in Merlo and Wilson (1995), where the set of admissible agreements and the bargaining protocol at each time follow a Markov process. Unfortunately our analysis cannot build on theirs since their characterization of equilibria relies on the assumption that, at all states, agents can choose an agreement from a standard bargaining set; this assumption fails in our game because in claim states there is a unique feasible termination.

4 Equilibria when strength is known

Assume that transition probabilities are known and remain constant $q_{it} = q_i, p_{it} = p_i$ for all t, and let $0 < q_{it}, p_{it} < 1, i = 1, 2$ so that no state of the game is absorbing.

Equilibrium outcomes and payoffs depend on whether claims meet opposition or surrender. We will show that agreement necessarily prevails in bargaining states. Surrender may prevail under both claim states, in one but not in the other, or opposition may occur in both. We will prove that only the first and the third scenario exclude each other.[3] We say that i's claim c_i is relevant at a given equilibrium σ , if it pays i at least as much as one period of confrontation; that is $c_i \geq \delta \left(q_i v_i(s_{ci}) + (1 - q_i)\, v_i\right),$ where v_i denotes player i's average payoffs in bargaining states and $v_i(s_{ci})$ denotes her expected payoffs in state s_{ci}. Formally, the following condition assures that the claims of both players are relevant.[4]

RC : Claims are relevant, for $i = 1, 2$,

$$c_i \geq \max\left\{\frac{(1 - q_i)\delta}{1 - \delta q_i}, \frac{1}{2}\right\}. \tag{1}$$

We start examining how the prevalence of agreement/surrender is linked across states. Observe than in equilibrium, the following must hold :

1. If surrender prevails in claim state s_{ci}, then agreement is reached in bargaining state s_{bj}.

2. If surrender prevails in states s_{c1} and s_{c2}, then agreement prevails in states s_{b1} and s_{b2}.

[3]Consequently, the uniqueness of stationary equilibria - a standard feature in bargaining games of alternating proposals - is not assured in asymmetric environments.

[4]$c_i \geq \frac{\delta(1-q_i)}{1-\delta q_i}$ implies that $c_i \geq \delta(q_i c_i + (1 - q_i)v_i)$ for all $v_i \leq 1$; that is, a share c_i dominates the payoff from continuation even if the payoff in the bargaining state is 1.

To understand the first observation, note that, in a bargaining state with 2 as proposer, a disagreement would prevail if and only if 1 preferred to reject any share that 2 were willing to propose; that is if $\delta\left(p_1 v_1+(1-p_1)c_1\right) \geq 1-\delta(p_1 v_2+(1-p_1)(1-c_1))$, or equivalently $p_1(v_1+v_2)+(1-p_1) \geq \frac{1}{\delta}$, but the latter inequality cannot hold since $v_1+v_2 \leq 1$ in any equilibrium. Similarly if surrender prevails in s_{c2}, then disagreement cannot prevail at s_{b1}. Our second observation follows immediately from the first.

We therefore conclude that confrontations cannot occur only in bargaining states. If there is confrontation in equilibrium, it must occur in a claim state. Is opposition possible in claim states, or do claims always induce surrender? A claim c_i cannot induce surrender when the share obtained upon surrender, $1-c_i$, is bounded above by the expected gains of an additional period of confrontation. That is, when

$$1-c_i<\delta\left(q_i v_j(s_{ci})+\left(1-q_i\right)v_j\right), \tag{2}$$

where $v_j(s_{ci})$ denotes j's expected payoffs in s_{ci}. If (2) holds opposition prevails, so expected payoffs $v_j(s_{ci})$ and $v_i(s_{ci})$ must solve $v_j(s_{ci})=\delta\left(q_i v_j(s_{ci})+\left(1-q_i\right)v_j\right)$ and $v_i(s_{ci})=\delta\left(q_i v_i(s_{ci})+\left(1-q_i\right)v_i\right)$; therefore $v_j(s_{ci})=\frac{(1-q_i)\delta v_j}{1-\delta q_i}$ and $v_i(s_{ci})=\frac{(1-q_i)\delta v_i}{1-\delta q_i}$. Substituting the expected payoffs in (2), we obtain that the necessary and sufficient condition for surrender at s_{ci} is

$$1-c_i \geq \frac{(1-q_i)\delta v_j}{1-\delta q_i}. \tag{3}$$

Writing

$$\phi_i\left(v_j\right) \equiv \frac{1-c_i-\delta v_j}{\delta\left(1-c_i-v_j\right)}, \tag{4}$$

condition (3) is equivalent to $q_i \geq \phi_i(v_j)$; in other words, the claim "must not be merely transient,"[5] it must persist at least with probability $\phi_i(v_j)$.[6] Then it follows (see the Appendix for a detailed argument) that confrontation prevails only in claim states. This completes the proof of our first proposition:

Proposition 1. Assume **RC**. In equilibrium the following hold:

1. Persistent claims award advantage: in claim states, player j surrenders and the split $(c_i, 1-c_i)$ is imposed if and only if $q_i \geq \phi_i(v_j)$.

2. Confrontation only in claim states: Termination, by agreement or surrender, is immediate unless a claim state occurs and $q_i \leq \phi_i(v_j)$.

Proposition 1 implies that any equilibrium profile must be one of the following three: a) A *confrontation* profile, where agreement is reached in bargaining states, and confrontation prevails otherwise. b) A *peaceful* profile, where agreement or surrender occurs at all states. c) An $i-$*advantage profile,* where agreement or surrender occurs in all states except s_{cj}.

The complete characterization of equilibria in general, non symmetric, environments involves straightforward but rather tedious algebra. We present it as Proposition 5 in the Appendix. Here we concentrate in symmetric environments; that is those satisfying:

SYM Strengths are symmetric; for $i = 1, 2$, $c_i = c$, $q_i = q$ and $p_i = p$.

Under SYM an equilibrium cannot be an $i-$advantage profile; and the equilibrium expected payoffs at bargaining states are $v = \frac{1}{2}$. Therefore, in claim states, surrender

[5]'If the enemy is to be coerced you must put him in a situation that is even more unpleasant than the sacrifice that you call on him to make. The hardships of that situation must not of course be merely transient. Otherwise the enemy would not give in but would wait for things to improve" (Clausewitz 1976, p. 77).

[6]Note that $\phi_i(v_j)$ is strictly increasing in v_j and satisfies $0 < \phi_i(v_j) < 1$, if and only if $\delta v_j \geq 1 - a_i$.

prevails if and only if $q \geq \phi(\frac{1}{2}) = \frac{\delta/2-(1-c)}{\delta(c-1/2)}$. Then a unique equilibrium exists and it is easily described:

Proposition 2. Equilibrium in symmetric environments with complete information. Under **RC** and **SYM** there is a unique equilibrium.

1. When $q \geq \frac{\delta/2-(1-c)}{\delta(c-1/2)}$, the equilibrium is a peaceful profile: In state s_{bi}, i offers to j a share $\delta\left[p\frac{1}{2}+(1-p)c\right]$ and she accepts; in state s_{ci}, j surrenders and i's claim of share c is imposed.

2. When $q < \frac{\delta/2-(1-c)}{\delta(c-1/2)}$, the equilibrium is a confrontation profile: In state s_{bi}, i offers to j a share $\frac{\delta}{2}\left[p+(1-p)\frac{(1-q)\delta}{(1-\delta q)}\right]$ and she accepts; in state s_{ci}, confrontation prevails.

We may now summarize the findings of this section. The prospect of advantage alters the distribution of the surplus because it may increase the expected gains of the agent engaging into confrontation. But this force is limited: when claims are excessive, in equilibrium, they are met with opposition and thus players prefer to avoid them when they are in a bargaining state. As long as the game starts in a bargaining state, an agreement prevails immediately and claims are never raised. Still, if the game begins in a claim state and persistence is relatively low, there is confrontation until the claim is dismissed and a bargaining state arises. In a nutshell, under complete information, from a bargaining state "one would never need to use the physical impact of the fighting forces - comparing figures and their strength would be enough."(Clausewitz 1976, p. 76.).

The fact is, however, that in real conflicts parties do resort to confrontation attempting to raise claims, and expecting that these will be imposed quickly. The lesson of history, moreover, is one of many wars where claims met resistance and were reversed. In the summer of 480 BC Xerxes was about to impose his advantage over Athens: the Persian alliance with Carthage assured control over the Greek colonies in Sicily and many of the smaller Greek states were eager to settle peacefully. Still Athens refused to yield and fortunes were reversed at Salamis. Napoleon's Russian campaign of 1812 successfully reached Moscow, yet it failed "because the Russian government kept its nerve and the people remained loyal and steadfast" (Clausewitz (1976), p. 628). In the summer of 1940 Hitler was celebrating victory and awaiting Churchill to sue for peace; he did not, and events took a very different course. Argentina's invasion of the Falkland in April of 1982 and Iraq's invasion of Kuwait in 1990 also fall under this category. Examples - as Korea or Vietnam - where parties alternate claims but none is imposed at the end are not unusual either.

Why would a rational agent engage in confrontation to attempt establishing a claim that might eventually prove so disadvantageous? We give an answer this question in the next section, as we extend our model to account for uncertainty and asymmetric information.

5 Uncertainty and Asymmetric Information

According to Blainey a common trait of wars is that the two parties "were persuaded to fight because most of their leaders were excessively optimistic and impatient men, and persuaded to cease fighting because those leaders, having failed, were replaced by more cautious men." (Blainey (1988) p. 123). We propose a formal set up in which Blainey's description holds precisely. We extend our model to address situations with uncertainty and asymmetric information and show that in these circumstances, confrontations might arise (and persist) along the equilibrium path even if the initial state is a bargaining state.

Our basic assumption is that the probabilities by which players sustain the commitment to their claims are unknown a priori; they have a random value that is realized only after claims are established. In a bargaining state, when a player considers whether to reject an offer, she has some private information about the strength of her potential claim, but she learns the precise value of this strength only if, and after, the claim is established. We assume that there are two types of players, that we name *hostile* and *lenient*, and that types are private information. A hostile type draws the persistence of her claims from a distribution biased towards high values; since she expects that her claim will be highly persistent and induce the opponent's surrender, she is more inclined to engage in confrontation and consequently is more demanding in bargaining states. A lenient type, expecting that her claims will have low persistence and induce opposition, is not inclined to confrontation and therefore accommodates to lower offers.

An offer is separating if it is acceptable to a lenient opponent but unacceptable to a hostile opponent; a pooling offer is one acceptable by both types. Suppose that the proposer makes a separating offer; this triggers war if the responder turns out to be hostile. Running the risk of war may well be ex-ante optimal (vis à vis to a pooling offer) when the probability of facing a lenient opponent is high enough. Hence separating offers are an equilibrium phenomenon when the proposer is an optimist - her prior beliefs assign high probability to the opponent being lenient - opening the door to war if the responder is an hostile type. Confrontation leads to a claim state with positive probability; upon this event the persistence of the claim is learned. If such persistence turns out to be high, the (initial) proposer must surrender. But even a hostile type, that ex ante expects a high persistence, may get a low draw. Then both players realize that reality has not matched their (rational) expectations, but they are stuck in confrontation until the claim is dismissed.

We will consider a symmetric environment where q_1 and q_2 are random variables whose value is realized only after the respective claim is established. After a player rejects a proposal and her claim state occurs, the probability to defend the claim it is publicly observed and it remains constant over time as long as the game remains in the same state; if the game returns to a bargaining state, future realizations of q_i are drawn independently. We assume that agents have private information on the distribution function of their own q_i. Each period that the game is in a bargaining state, players privately observe their type (the distribution of q_i) for that period. Formally we assume:

U1 $c_1 = c_2 = c > \frac{1}{2}$ and $p_1 = p_2 = p$, and these are known.

U2 There are two types of players, $\tau_i \in \{l, h\}$, at each $s_t = s_{bj}$ types are drawn independently with probability

$$\Pr(\tau_i = l \mid s_t = s_{bj}) = \lambda, 0 < \lambda < 1,$$

and realized types are private information.

U3 q_i is drawn with distribution F^{τ_i} at each $s_t = s_{ci}$ such that $s_{t-1} = s_{bj}$; then it becomes public and remains constant as long as the claim state persists.

U4 F^h first order stochastically dominates F^l: $F^h(q) \leq F^l(q)$ for all $q \in [0, 1]$.

Assumptions **U1**, **U2** and **U3** are for simplicity. The first two could be relaxed (at non-negligible notational and expositional cost) to account for larger type sets, asymmetries and uncertainty on p and c, or serial correlation. Assumption **U3** could also be relaxed to allow that i retains some information advantage on q_i after state s_{ci} is realized, assuming that agents learn their own q_i quickly while opponents must learn it by the evidence that the claim persists. This extension would deliver equilibrium histories where agents initially oppose claims but eventually surrender. **U4** is the crucial assumption.

A *system of beliefs* for player i, π_i, maps histories into probability distributions over the types of player j. A perfect Bayesian equilibrium is an assessment (σ, π) such that, σ is a pair of strategies that are best response to each other at each history, and π is a belief profile consistent with Bayes' rule. At a Markov strategy actions depend only on the state, the current beliefs and the current offer. Henceforth the term equilibrium refers to perfect Bayesian equilibrium in Markov strategies. Observe that an equilibrium is fully characterized by specifying proposals and acceptance/surrender thresholds for each type at each state. Note that the system of beliefs does not need to be specified beyond Bayes Rule because, at any off the equilibrium history that is not terminal, either a claim state is attained and q is fully revealed; or the game remains in the bargaining state and new types are drawn - in which case players must believe that their opponent is lenient with probability λ.

Fix an equilibrium and let v denote the ex-ante (before types are drawn) expected gains of players in bargaining states. (Note that, if rejections occur with positive probability, $v < \frac{1}{2}$). Observe that in state s_{ci} if the probability that i maintains her claim one more period is sufficiently small disagreement prevails: Indeed, by the same argument used to prove Proposition 1 we know that if state s_{ci} occurs the game stays in confrontation as long as $q_i \leq \widehat{q}$, where $\widehat{q} = \phi(v)$ (recall that $\phi(v)$ is determined by equation (4)). Otherwise, for $q_i > \widehat{q}$, player j surrenders and the split $(c, 1-c)$ is imposed.[7]

We will see that the equilibrium is pooling - so that immediate agreement prevails in bargaining because the first proposal is accepted regardless of the responder's type -

[7]Note that $0 < \widehat{q} < 1$ only if $c + \delta v > 1$.

if and only if the probability that players are of lenient type is not too high. Otherwise, the equilibrium is separating. Along the separating equilibrium play proceeds as follows. Player i makes an offer that leaves j indifferent between acceptance and rejection if she is lenient (and that a hostile j strictly prefers to reject). Upon rejection, with probability p a new round of bargaining follows whereas the game enters state s_{cj} with probability $1-p$. If state s_{cj} is realized, then the value of q_j is observed by both parties. Immediate surrender to shares $(c, 1-c)$ follows if and only if q_j satisfies $q_j \geq \widehat{q}$; otherwise confrontation follows until the claim is dismissed. At a new bargaining state, the players observe their new type, a new separating offer is made, and so on.

For a given the equilibrium with ex-ante expected gains v, we let $r^l(v)$ and $r^h(v)$ denote the responder's expected gains upon rejection in state s_{bi}, respectively for a lenient and a hostile responder. Also, $R^h(v)$ denotes the proposer's continuation value when her opponent rejects and the updated belief induced by rejection assigns probability 1 to the hostile type. Now, define λ_v as the ratio:

$$\lambda_v \equiv \frac{1 - r^h(v) - R^h(v)}{1 - r^l(v) - R^h(v)}, \tag{5}$$

and we will refer to $\lambda_{v=1/2}$ as the *optimism threshold.*

We are now ready to describe the unique equilibrium:

Proposition 3. Equilibrium in symmetric environments with private information. Under assumptions **U1** to **U4** there is a unique equilibrium outcome.

1. If the ex-ante probability of facing a lenient type exceeds the optimism threshold, that is $\lambda > \lambda_{1/2}$, then the equilibrium is separating. The expected gains

are $v = v^*$, where v^* is unique solution to

$$v = \lambda \frac{1}{2} + (1-\lambda) \frac{R^h(v) + r^h(v)}{2}. \tag{6}$$

For all t, in state s_{bi}, i offers $x^* = r^l(v^*)$ and j accepts only if $\tau_j = l$. If $\tau_j = h$ and $s_{t+1} = s_{cj}$ player i surrenders granting c to j if $q_j \geq \phi(v^*)$; for $q_j < \phi(v^*(\lambda))$ confrontation prevails at all $t+k$ until $s_{t+k} = s_{bi}$.

2. Otherwise, the equilibrium is pooling. The expected gains are $v = 1/2$, and for all t, in state s_{bi}, i offers $y^* = r^h(\frac{1}{2})$ and j accepts.

The proof is in the Appendix.

In bargaining states war starts because the proposer's optimism, expecting a lenient responder with high probability, is not confirmed; it continues in claim states because the realized persistence disappoints the the optimist belief of the hostile type. In summary, war occurs when reality disappoints the proposer's optimism; and persist longer when both agents have optimist (rational) expectations that are not realized ex-post.

Equilibrium histories. It is now immediate that when $\lambda > \lambda_{1/2}$, so that a separating equilibrium prevails, a great variety of equilibrium histories are possible, including prolonged confrontation and fortune reversals. Three categories of wars are possible:

1. Useless war: An agreement $(\frac{1}{2}, \frac{1}{2})$ does not prevails until $t = k > 1$. Prior to agreement $k-1$ proposals meet rejection (because a hostile responder is drawn) but neither side ever raised a claim, at $t = k$ the responder type is lenient and agreement prevails.

2. War with victory: A concession to $(c, 1-c)$ prevails at $t = k+1$. After sequence of $k \geq 1$ rejections in the bargaining state the game enters a claim

state at $k+1$, then upon the observation that $q \geq \hat{q}$ the opponent surrenders.

3. FORTUNE REVERSAL: War prevails at least for $t = k+n$ periods; from $t = 0$ to $k \geq 1$ proposals meet rejection, a claim is established at $t = k+1, q$ is low, the claim is opposed and it persists for n periods. The continuation might lead to immediate agreement or to any history of type 1,2 or 3.

The assumption that q becomes public immediately when the claim state is realized rules out equilibria where claims are initially challenged but eventually prevail. Such histories might occur in equilibrium under the assumption that agents learn their own q quickly while opponents must learn it by the evidence that the claim persists.

For realizations of the parameters that deliver a separating equilibrium, the probability of an immediate agreement is λ; and the probability of a useless confrontation is $(1-\lambda)(1-p)$ each period. To evaluate the ex-ante likelihood of victories and fortune reversals we must measure $\Pr(q \geq \hat{q})$, which requires an explicit computation of the equilibrium. We do this next.

5.1 Equilibrium Computation and Comparative Statics

An explicit computation of the equilibrium requires a specification for $F^l(q)$ and $F^h(q)$. For simplicity, we postulate a degenerate distribution $F^l(q)$ and a two point distribution $F^h(q)$; the lenient type draws a low $q < 1/2$ whith probability 1, and the hostile type draws the low q with prob α and a high $q = 1$, with prob $1-\alpha$.

What increases the probability that war starts?

War occurs with positive probability only when the optimism threshold is low enough, that is $\lambda_{1/2} < \lambda < 1$; and when this inequality holds, the probability that war starts is $1-\lambda > 0$ at every period that the game spends in the bargaining state.

Consequently, to evaluate how changes in the environment translate into changes on the probability that war starts, we must examine $\lambda_{1/2} = \frac{1-r^h(1/2)-R^h(1/2)}{1-r^l(1/2)-R^h(1/2)}$, as a function of the parameters δ, p, c, α.

The values of $1-p$ and α measure the strength of players upon confrontation; greater values translate in increases of the agents' continuation values upon confrontation. Consequently, $\frac{\partial \lambda_{1/2}}{\partial p} \geq 0$, $\frac{\partial \lambda_{1/2}}{\partial \alpha} \leq 0$, with strict inequality whenever $\lambda_{1/2} < 1$. Evaluating $\frac{\partial \lambda_{1/2}}{\partial \delta}$ and $\frac{\partial \lambda_{1/2}}{\partial c}$ requires some algebra, but it is not hard to check that, $\lambda_{1/2}$ is strictly decreasing both in δ and in c. Figures 1 and 2 display $\lambda_{1/2}$ as a function of δ and c for α and the low q fixed at 1/2.

We summarize these observations in the following proposition:

Proposition 4. The optimism threshold threshold $\lambda_{1/2}$ decreases, and so the separating equilibrium (and war) prevails for a wider range of ex-ante belief λ, whenever one of the following changes takes place:

1. Claims become more extreme.

2. The claim state arises with greater probability.

3. The hostile type expects greater persistence of claims.

4. Per-period losses decrease.

For $F^l(q)$ and $F^h(q)$ fixed, the size of claims c, and the cost of confrontation $1-\delta$, have impact only over the threshold $\lambda_{1/2}$, and not on the effective probability of war $1-\lambda$. Therefore small changes in c, or δ are inconsequential. However, greater changes can either drop the probability of confrontation from $1-\lambda$ to 0, or bring it up form 0 to $1-\lambda$.

For example, let $\underline{q} = \alpha = 1/2$, $\lambda = .83$, $c = .8$, and $\delta = .8$. In this environment $\lambda_{1/2} = .79 < \lambda = .83 < 1$; so war occurs with probability $1-\lambda = .17$. A small

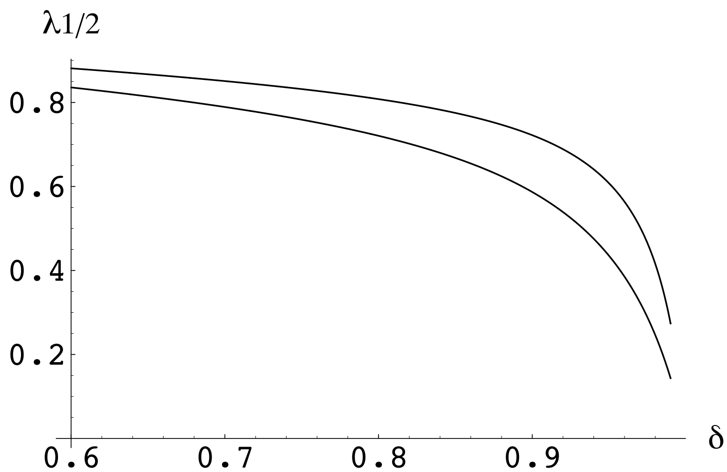

Figure 1: $\lambda_{1/2}$ as a function of δ for $c = .65$ and $.85$.

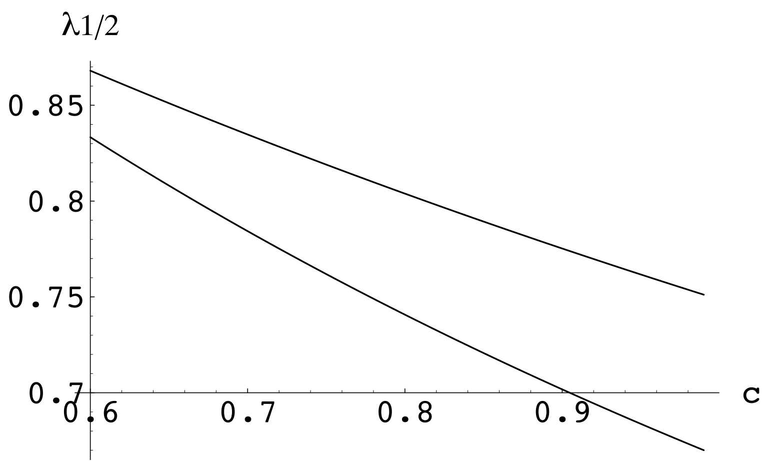

Figure 2: $\lambda_{1/2}$ as a function of c for $\delta = .7$ and $.8$.

change in δ; has an effect on $\lambda_{1/2}$ but, as long as the inequality $\lambda_{1/2} < \lambda = .83$ is maintained, it has no impact in the probability of war. However, with a change to $\delta' > .7$, the inequality is reversed, and with $\lambda_{1/2} > \lambda$ the probability of war drops to zero. Thus, an sufficient increase in the destruction of surplus caused by confrontation can eliminate the possibility of war.

Likewise, small changes in c have no impact in the probability of war but a sufficient decrease in c, say to $c' < .7$, drops the probability of war to zero. Interpreting c as the value of a special landmark, our results are consistent with a prevalent fact: war is triggered by a sudden increases the relative value of a landmark (say, the discovery of a new oil field).

Further analysis requires the explicit computation of the expected gains.

Separating Equilibrium values and the losses of war.

The difference between potential gains and expected payoffs in equilibrium measure of the losses imposed by war; the expected losses caused by war. Under a pooling equilibrium the ex-ante expected payoff is $v = 1/2$ for each agent and therefore no loss is incurred. In a separating equilibrium the expected loss is $1 - 2v > 0$.

In the appendix we establish that under the present specification

$$v = \frac{1}{2}\frac{\lambda + (1-\lambda)2g}{1-(1-\lambda)h},$$

where $h \equiv \delta\left(p + \frac{(1-p)\delta(1-q)\alpha}{1-\delta q}\right)$ and $g \equiv \frac{\delta(1-p)(1-\alpha)}{2}$. It is easily checked that $2g+h < 1$, and therefore $\frac{\partial v}{\partial \lambda} = \frac{1}{2}\frac{1-h-2g}{(1-h+h\lambda)^2} > 0$. Thus, the worse outcomes are attained at the lower values of λ that satisfy $\lambda > \lambda_{1/2}$. Figure 3 displays the expected gains as functions of λ for $c = .7, \alpha = p = q = .5$, and $\delta = .9$.

On the other hand, v increases in δ. Figure 4 displays the expected gains as a functions of δ for $c = .7, \alpha = p = q = .5$, and $\lambda = .7$.

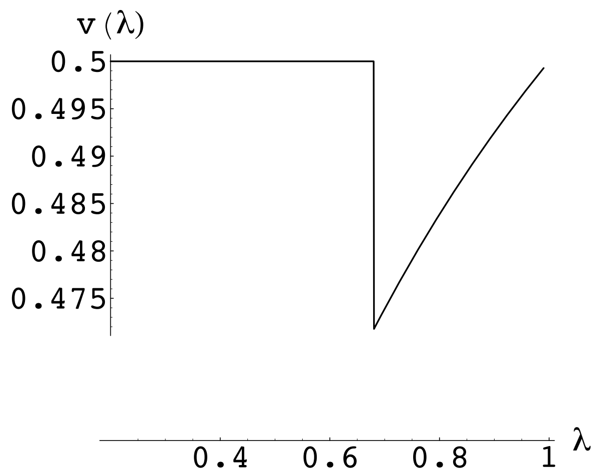

Figure 3: Expected gains as a function of λ, $\delta = .9$

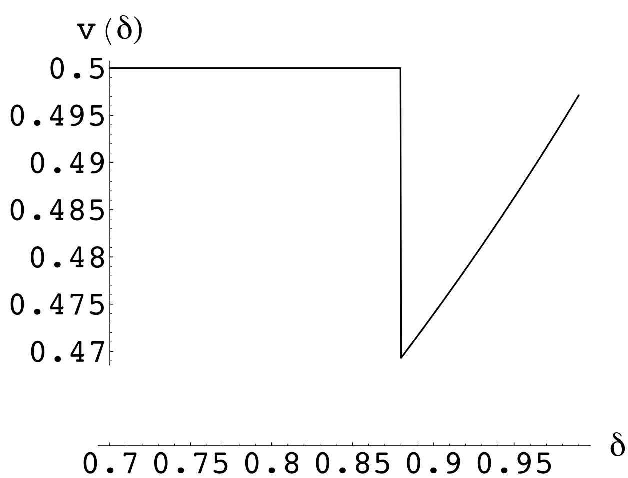

Figure 4: Expected gains as a function of δ

It is also immediate that v increases in p and decreases in q and α.

We emphasize that the qualitative features of this illustrative example are general. Thus we conclude that when $\lambda \geq \lambda_{1/2}$, other parameters fixed, $\Pr(q \geq \hat{q})$ is greater the lower λ, and then lower δ. This has two implications: First, that the victory of one side is more likely (i.e. $\Pr(q \geq \hat{q})$ is higher) when the probability of drawing a hostile type is greater. And second, that fortune reversals are more likely (i.e. $\Pr(q \geq \hat{q})$ is lower) when the cost of confrontation is large.

6 Final remarks

We have presented a model of bargaining through confrontation where the set of admissible agreements follows a Markov process. Our contribution points out that, when the ability to exercise commitment is linked to the use of force, contests to attain advantage entail uncertainties and asymmetries that may fuel prolonged episodes of confrontation.

Assuming that a the set of states has the minimal cardinality and that transitions governed by stationary probabilities, we have carried out our analysis in the simplest of the scenarios. Real conflicts have immense sets of states and their transition probabilities are hardly stationary. Nevertheless the qualitative nature of our results does not rely on our drastic simplifications. We hope to provide useful intuitions relevant in the analysis of real, more complex, disputes and guide a revised look at the empirical evidence.

The value of claims and the probabilities to establish and maintain them have been assumed exogenous. In reality, however, these probabilities depend on the degree of advantage aimed by a player; as well as on the opponent's strength and claim value. An extension of our model allowing that bargaining parameters are interrelated and endogenously determined by the strategic choice of the agents will be the object of further research.

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A Appendix

Proof of Proposition 1 2):

Proof. By equation (1), the responder payoffs in state s_{bi} are such that

$$\text{if } (q_1, q_2) \ll (\phi_1(v_2), \phi_2(v_1)), \quad v_i^r = \delta v_i \left(p_i + (1-p_i) \tfrac{(1-q_i)\delta}{1-\gamma q_i} \right),$$
$$\text{if } (q_1, q_2) \gg (\phi_1(v_2), \phi_2(v_1)), \quad v_i^r = \delta \left(p_i v_i + (1-p_i) a_i \right),$$

while the proposer obtains $v_j^p = 1 - v_i^r$.

Clearly agreement must prevail in a state s_{bi} when confrontation prevails in both claim states. Hence we only need to consider strategy profiles that yield concession in state s_{ci} but not in state s_{cj}.

Consider first profiles that yield concession in state s_{c1} but not in state s_{c2}. When 1 proposes, 2 accepts as long as her share is at least $v_2^r = \delta v_2 \left(p_2 + (1-p_2) \frac{(1-q_2)\gamma}{(1-\gamma q_2)} \right)$, and thus agreement can be attained if and only if 1 prefers to offer that share over disagreement. That is,

$$\delta v_1 \left(p_2 + (1-p_2) \frac{(1-q_2)\delta}{(1-\delta q_2)} \right) \leq 1 - \delta v_2 \left(p_2 + (1-p_2) \frac{(1-q_2)\delta}{(1-v q_2)} \right),$$

or equivalently $v_1 + v_2 \leq \frac{1-\delta q_2}{\delta(\delta(1-q_2)+(1-\delta)p_2)}$. A condition that always holds since the second term exceeds 1. □

Proof of proposition 3.

Proof. Fix an equilibrium. Given ex-ante expected gains at bargaining states v, the continuation values upon a rejection $r^l(v)$ and $r^h(v)$ are

$$r^l(v) = \delta \left(pv + (1-p) \left[v\Phi^l + (1 - F^l(\widehat{q}))c \right] \right),$$

$$r^h(v) = \delta \left(pv + (1-p) \left[v\Phi^h + (1 - F^h(\widehat{q}))c \right] \right).$$

where $\Phi^\tau \equiv \delta \int_0^{\widehat{q}} \frac{1-q}{1-\delta q} dF^\tau(q) dq$. It is a matter of simple algebra to check that $r^h(v) > r^l(v)$.[8]. Similarly, the proposer's continuation value upon responders rejection, when the induced beliefs are π is

$$R^\pi(v) = p\delta v + (1-p)\delta \left[v\Phi^\pi + (1 - F^\pi(\widehat{q}))(1-c) \right].$$

Recall that $\lambda_v \equiv \frac{1-r^h(v)-R^h(v)}{1-r^l(v)-R^h(v)}$, where R^h denotes R^π when belief π assigns probability 1 to the hostile type.

Next, we point out that at a pooling equilibrium agreement prevails for sure in the bargaining states. Assume for the sake of the argument that disagreement prevails for sure in state s_{bi}. It is then necessary that the proposer prefers disagreement to an agreement that the lenient responder accepts, that is $R^\pi(v) > 1 - r^l(v)$. However, at the hypothesized pooling equilibrium profile the beliefs of the proposer upon rejection are $G = \lambda F^l + (1-\lambda)F^h$ so that the continuation value is $R^g(v) = \delta p v + \delta(1-p) \left[v\Phi^g + (1 - G(\widehat{q}))(1-c) \right]$. It is then immediate to check that $v \leq \frac{1}{2}$ implies that $R^g(v) + r^l(v) < 1$, a contradiction. Consequently, an equilibrium must be either a pooling equilibrium where both types accept, or a separating equilibrium where the responder accepts if and only if she is lenient.

Let us now discuss the necessary and sufficient conditions for pooling or separating equilibria. Consider first a pooling equilibrium. Since in states s_{bi} the initial proposal is surely accepted, the complete symmetry of the environment implies that $v = \frac{1}{2}$. If state s_{ci} occurs (off the equilibrium path) confrontation prevails for $q < \phi(\frac{1}{2})$. Hence the responder's rejection values are uniquely given as $r^l(\frac{1}{2})$ and $r^h(\frac{1}{2})$. On the other hand, if a rejection were to reveal that the responder is hostile the proposer's rejection

[8] $r^h(v) - r^l(v) = (1-p)\delta \left[v\left(\Phi^h - \Phi^l\right) + c\left(F^l(\widehat{q}) - F^h(\widehat{q})\right) \right]$ and observe that the right hand side is positive if and only if $c \geq \delta v \frac{\Phi^l - \Phi^h}{F^l(\widehat{q}) - F^h(\widehat{q})}$, an inequality that holds, since $c \geq \frac{1}{2}$, $\delta v < \frac{1}{2}$ and $\frac{\Phi^l - \Phi^h}{F^l(\widehat{q}) - F^h(\widehat{q})} < 1$.

value would be $R^h(\frac{1}{2})$. By hypothesis, in state s_{bi}, both types of responder must accept, hence the proposer must offer $y^* = r^h(\frac{1}{2})$. At the alleged pooling equilibrium the proposer must prefer a sure payoff $1 - y^*$ over making a lower offer, $y' < y^*$, and getting acceptance only if the responder is lenient. The least that must be offered to obtain acceptance with positive probability is $y' = r^l(\frac{1}{2})$, hence it is necessary that $1 - r^h(\frac{1}{2}) \geq \lambda(1 - r^l(\frac{1}{2})) + (1-\lambda)R^h(\frac{1}{2})$; that is, the prior probability of the lenient type must not exceed the optimism threshold at $v = \frac{1}{2}, \lambda \leq \lambda_{\frac{1}{2}}$. In addition to necessary, this inequality is also sufficient for the existence of a pooling equilibrium: It suffices that the belief of the proposer upon (off the equilibrium) rejection assigns probability 1 to the hostile type.

Next, consider a separating equilibrium. Note that $v < \frac{1}{2}$, because in state s_{bi} the proposer offers only the rejection value of the lenient type, $x = r^l(v)$, which is accepted by l but not by h. Moreover, the proposers's beliefs about q upon rejection must be F^h so that the proposer rejection value is $R^h(v)$. Since either player proposes with equal probability, v must satisfy $v = \frac{1}{2}\left(\lambda r^l(v) + (1-\lambda)r^h(v)\right) + \frac{1}{2}\left(\lambda(1 - r^l(v)) + (1-\lambda)R^h(v)\right)$, that simplifies to

$$v = \lambda\frac{1}{2} + (1-\lambda)\frac{R^h(v) + r^h(v)}{2},$$

which is equation (6). Denote the solution to (6) by v^*. This solution always exists, and it is unique. (It is immediate to check that $\gamma : [0,1] \to [0,1]$, where $\gamma(v) = v - \lambda\frac{1}{2} - (1-\lambda)\frac{R^h(v)+r^h(v)}{2}$, is a contraction and therefore has a unique fixed point.) Furthermore, note that $0 < v^* < \frac{1}{2}$. In state s_{bi}, i must offer $r^l(v^*)$; and this must be preferred to offering $r^h(v^*)$, i.e.

$$\lambda \geq \lambda_{v^*}. \tag{7}$$

Therefore the necessary and sufficient condition for the existence of a separating equilibrium is that $\lambda \geq \lambda_{1/2}$: since $v^* < \frac{1}{2}$ and λ_v is strictly increasing in v, $\lambda > \lambda_{\frac{1}{2}}$

implies that $\lambda \geq \lambda_{v^*}$. This completes the proof. $\square$

Computation of Equilibrium values

In a separating equilibrium, the ex-ante expected payoff v, must solve equation (6), i.e.$v = \lambda/2 + (1-\lambda)\frac{r^h(v)+R^h(v)}{2}$. It is immediate to check that, under the present specification for $F^l(q)$ and $F^h(q)$,

$$r^h(v) = \delta\left[pv + (1-p)\left(\alpha\frac{v\delta(1-q)}{1-\delta q} + (1-\alpha)c\right)\right],$$

and

$$R^h(v) = \delta\left[pv + (1-p)\left(\alpha\frac{v\delta(1-q)}{1-\delta q} + (1-\alpha)(1-c)\right)\right];$$

so that

$$\frac{r^h(v)+R^h(v)}{2} = \delta v\left[p + (1-p)\alpha\frac{\delta(1-q)}{1-\delta q}\right] + \delta(1-p)(1-\alpha)\frac{1}{2}.$$

Writting $h \equiv \delta\left(p + \frac{(1-p)\delta(1-q)\alpha}{1-\delta q}\right)$ and $g \equiv \frac{\delta(1-p)(1-\alpha)}{2}$, v must solve $v = \frac{\lambda}{2} + (1-\lambda)(hv+g)$. Hence

$$v = \frac{1}{2}\frac{\lambda + (1-\lambda)2g}{1-(1-\lambda)h}.$$

It is easily checked that $2g + h < 1$, and therefore $\frac{\partial v}{\partial \lambda} = \frac{1}{2}\frac{1-h-2g}{(1-h+h\lambda)^2} > 0$. Derivatives with respect to other parameters are immediate.

Proposition 5. Equilibria in non-symmetric, complete information, environments: Under **RC** an equilibrium always exists. Moreover,

1. A peaceful equilibrium excludes the existence of a confrontation equilibrium. Equilibria in 1−advantage strategies and 2−advantage strategies may coexist; and these may coexist with either a peaceful equilibrium or a confrontation equilibrium.

2. The expected payoffs of player i in a bargaining state under the four categories of (potential) equilibrium profiles are:

$v_i(\sigma^p)$	$v_i(\sigma^i)$	$v_i(\sigma^j)$	$v_i(\sigma^c)$
$\frac{1-\delta p_j+\delta c_i(1-p_i)-\delta c_j(1-p_j)}{2-\delta(p_i+p_j)}$	$\frac{\rho_j}{\rho_j+\lambda_i}$	$\frac{\lambda_j}{\rho_i+\lambda_j}$	$\frac{\rho_j}{\rho_2+\rho_1}$

where $\rho_i \equiv 1-\delta p_i-(1-p_i)\frac{(1-q_i)\delta^2}{1-\delta q_i}$ and $\lambda_i \equiv 1-\delta(p_i+c_i(1-p_i))$.

Remark. The multiplicity of stationary equilibria opens the door to subgame perfect equilibria in which confrontation occurs in the bargaining state, provided that non-stationary strategies are allowed. Since this is a standard result we do not elaborate it further.

Proof. The values of expected payoffs at bargaining states for each of the potential equilibrium strategy profiles follows from straightforward algebra. It is also immediate to check that $v_i^i > \max\{v_i^c, v_i^p\} > \min\{v_i^c, v_i^p\} > v_i^j$.

Given a configuration of parameters $(c_{1,}c_2, p_1, p_2)$ define the sets

$$Q^c = \{(q_1,q_2) \mid (q_1,q_2) \ll (\phi_1(v_2^c), \phi_2(v_1^c))\},$$
$$Q^p = \{(q_1,q_2) \mid (q_1,q_2) \gg (\phi_1(v_2^p), \phi_2(v_1^p))\},$$
$$Q^i = \left\{(q_1,q_2) \mid q_i > \phi_i\left(v_j^i\right), q_j \leq \phi_j\left(v_i^i\right)\right\}.$$

Necessary and sufficient conditions to sustain each of the potential profiles as an equilibrium are now immediate:

1. A peaceful equilibrium exists if and only if $(q_1,q_2) \in Q^p$;

2. A confrontation equilibrium exits if and only if $(q_1,q_2) \in Q^c$;

3. An $i-$advantage equilibrium exits if and only if $(q_1,q_2) \in Q^i$.

Therefore what profiles can prevail as equilibria for each parameter configuration depends on the specific geometry of Q^c,Q^p, Q^1 and Q^2. Consider first the set Q^p since it is specially simple: Since v_i^p is independent of (q_1, q_2), so is $\phi_i(v_j)$ and consequently

$$Q^p = \{(q_1, q_2) \mid (q_1, q_2) \gg (\widehat{q}_1, \widehat{q}_2)\}\,, \tag{8}$$

where $\widehat{q}_i = \phi_i(v_j^p)$.

On the other hand, observe that Q^c can be expressed as

$$Q^c = \{(q_1, q_2) \mid q_1 \leq \varphi_1(q_2), q_2 \leq \varphi_2(q_1)\}\,, \tag{9}$$

where $y = \varphi_1(q_2)$ if and only if y solves $y = \phi_1(\frac{\rho_1(y)}{\rho_2+\rho_1(y)})$, where $\rho_i(y) \equiv 1 - \delta p_i - (1-p_i)\frac{(1-y)\delta^2}{1-\delta y}\rho_i$; and analogously for $\varphi_2(q_1)$. It is straightforward to check that the functions φ_i are decreasing.

Note that $Q^c \cap Q^p = \emptyset$. Indeed, since both φ_i are decreasing it is straightforward to check that $\widehat{q}_1 > \varphi_2(\widehat{q}_2)$ and $\widehat{q}_2 > \varphi_1(\widehat{q}_1)$.

With respect to Q^i observe that $(v_i^i, v_j^i) = (\frac{\rho_j}{\rho_j+\lambda_i}, \frac{\alpha_i}{\rho_j+\lambda_i})$ depends only on c_i and q_j. Hence

$$Q^i = \left\{(q_1, q_2) \mid q_i > \psi_i(q_j), q_j \leq \overline{q}_j\right\}, \tag{10}$$

where $\overline{q}_j$ solves $q_j = \phi_j\left(v_i^i\right) = \frac{1-c_j-\delta\frac{\rho_j}{\rho_j+\lambda_i}}{\delta\left(1-c_j-\frac{\rho_j}{\rho_j+\lambda_j}\right)}$ and $\psi_i(q_j)$ is $\phi_i\left(v_j^i\right) = \frac{1-c_i-\delta\frac{\lambda_i}{\rho_j+\lambda_i}}{\delta\left(1-c_i-\frac{\lambda_i}{\rho_j+\lambda_i}\right)}$. Since $v_i^i > v_i^p$ we obtain that $\overline{q}_j > \widehat{q}_j$. Moreover, since ϕ_i is increasing, $\psi_i(q_j)$ decreases in q_j and furthermore $\varphi_i(q_j) > \psi_i(q_j)$.

We have thus shown that $Q^i \cap Q^p \neq \emptyset$, $Q^i \cap Q^c \neq \emptyset$ and $q \notin Q^p \cup Q^c \Leftrightarrow q \in Q^1 \cup Q^2$. Hence, an equilibrium always exists, it is generally not unique since different types of equilibria (up to three) may coexist for some parameter configurations; yet a peaceful equilibrium and an confrontation equilibrium never coexist. □